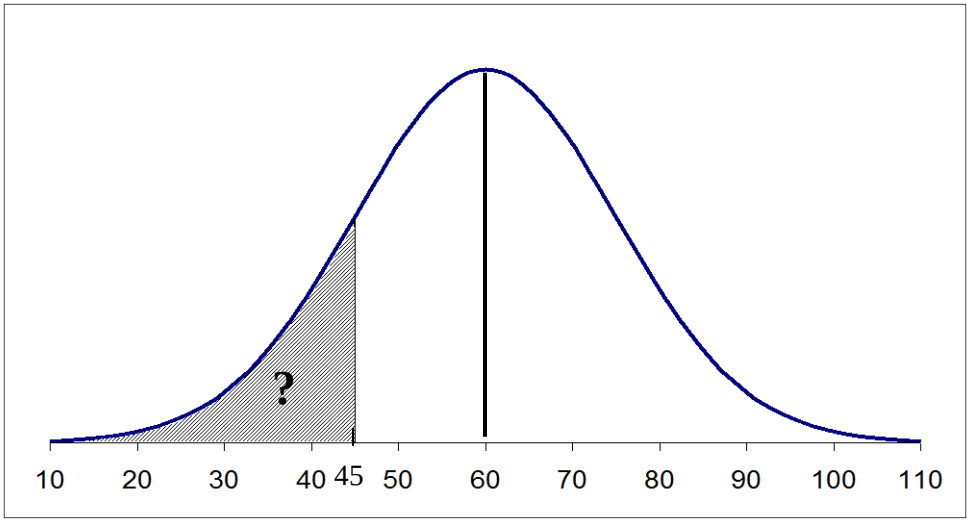
Solving Problems Involving Using Normal Distribution

Problem 1: Suppose that the data concerning the first-year salaries of Baruch graduates is normally distributed with the population mean μ = \$60000 and the population standard deviation σ = \$15000. Find the probability of a randomly selected Baruch graduate earning less than \$45000 annually. To answer this question, we have to find the portion of the area under the normal curve from 45 all the way to the left.



To find this area, you can apply complex mathematical formulas, or you can use the Z-table, in which statisticians have already applied those formulas for you. Because they could not develop tables for every possible combination of the mean and the standard deviation, statisticians developed one standardized and simplified normal distribution with the mean of 0 and the standard deviation of 1.

All other distributions with different : and σ can be converted into a standardized normal distribution using the **transformation formula**:

$$Z = \frac{X - \mu}{\sigma}$$

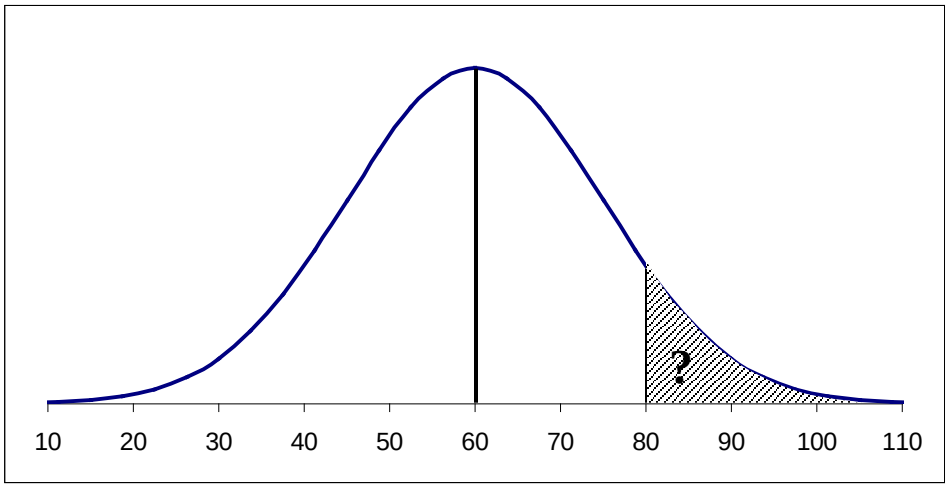
Think about this transformation as of a ruler: for example, if you know your height in inches, you can use a formula to convert it into centimeters.

Using the transformation formula, find the value of Z and then find the number that corresponds to that Z in the body of Z-table:

$$Z = \frac{45 - 60}{15} = -1.$$

The table value that corresponds to –1 is 0.1587. This number indicates the area under the curve from 45 all the way to the left. It also indicates that the probability of randomly selecting a Baruch graduate who makes less than $45000 a year is 15.87%.

Problem 2: Now, keeping all of the above in mind, find the probability of randomly selecting a Baruch graduate that makes more than $80000 a year, given the same normal distribution.



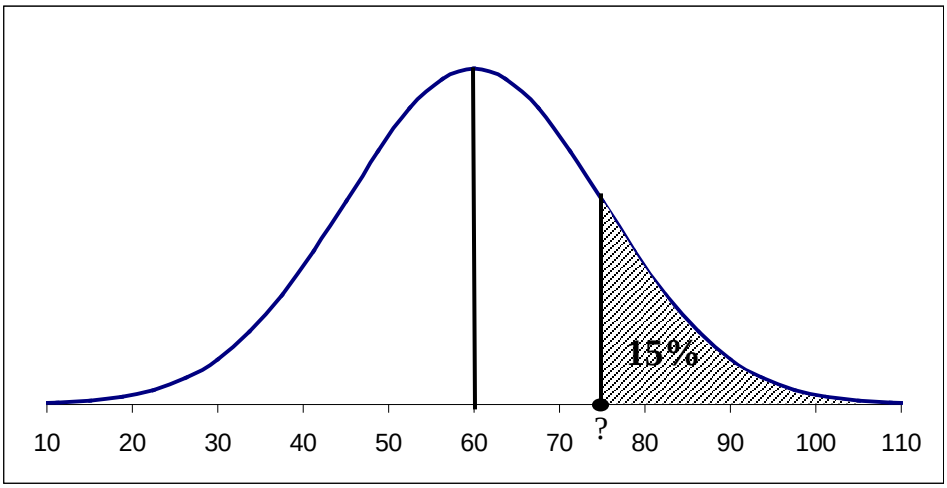
Find the Z-value using the transformation formula:

$$Z = \frac{80 - 60}{15} = 1.33$$

The table value that corresponds to Z = 1.33 is 0.9082 or 90.82%. This is not the final answer, however, because as you can see, the Z-table only shows the values *less than (and to the left of) each value of Z*. 90.82% that we found in the table is the probability of randomly selecting a Baruch graduate that earns less than $80000 annually. If you remember that the entire normal curve covers 100% of the distribution, you will be able to find the complement probability or the area under the curve to the right of 80. Just subtract the table value from 100%:

$$P(X>80) = 100\% - 90.82\% = 9.18\%$$

Problem 3: Find the range of annual salaries of the top 15% earners, given the same distribution of Baruch graduates.



In this problem, the situation is reversed. You are given an area under the curve and have to find a specific value of X that would correspond to the annual salary that separates the highest-paid 15% of Baruch graduates. In this situation, you have to move backwards. First, find 85% (or 0.8500) in the body of the Z-table. Again, you use 85% (100% - 15%) because the Z-table only works for the areas below each value of Z. Go across and up to the margins of the table to find the value of Z. That value is 1.04. Now you have all the data to use the transformation formula and solve for X:

$$Z = \frac{X - \mu}{\sigma}$$

$$1.04 = \frac{X - 60}{15}$$

$$1.04 \times 15 = X - 60$$

$$X = 15.6 + 60 = 75.6$$

Thus, the highest-paid 15% of recent Baruch graduates make $75600 and above per year.

To summarize the problems so far, here are the two cases that you will encounter in all of the problems dealing with normal distribution. Compare what is given, what you need to find in both cases, and the strategies to solve both problems.

	Given:	***Need to find:***	***Strategy:***
Case 1:	*Value(s) of X* *Mean* *Standard deviation*	*Probability or area under the curve*	1. *Use transformation formula to calculate Z* 2. *Find Z on the margins of Z-table* 3. *Find the corresponding area under the curve*
Case 2:	*Probability or area* *Mean* *Standard deviation*	*Value(s) of X*	1. *Locate the area in the body of the Z-table* 2. *Find corresponding Z on the margins of Z-table* 3. *Use transformation formula and solve for X*