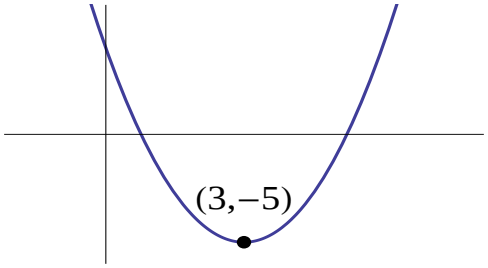
MATH 2003
TEST F
This part of the exam is to be done without a calculator.

1. Find the slope and $y-$intercept (m and b respectively) of the line which passes through $(-3, 5)$ and is perpendicular to $y = \frac{1}{3}x - 2$.

   (A) $m = \frac{1}{3}, b = 6$ (B) $m = -3, b = -4$ (C) $m = -\frac{1}{3}, b = 4$

   (D) $m = -3, b = 14$ (E) $m = 3, b = 14$

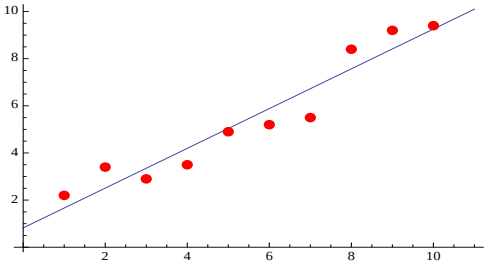
2. The graph of a quadratic function of the form $f(x) = x^2 + bx + c$ is shown. What is the value of b?



   (A) 1 (B) 3 (C) -5

   (D) -6 (E) 6

3. A market research study has collected data points which indicate that the relationship between the price of a certain commodity and the quantity supplied is nearly linear. The data is graphed below with price (in dollars) along the y-axis and quantity supplied (in millions) along the x-axis along with a best fit line. Which of the following values is the most reasonable for the linear correlation coefficient?



   (A) $r = -.183$ (B) $r = .084$ (C) $r = 1.3$

   (D) $r = -.984$ (E) $r = .967$

4. Let $f(x) = \frac{1}{x^2-1}$ and $g(x) = \frac{1}{x-2}$. What is the domain of $f(g(x))$?

(A) All $x \neq -1, 1, 2$ (B) All $x \neq -1, 1, 3$ (C) All $x \neq 0$

(D) All $x \neq -1, 0, 1, 2$ (E) All $x \neq 1, 2, 3$

5. Consider the system of equations: $\begin{cases} x - 3y = 2 \\ kx + 9y = 1 \end{cases}$

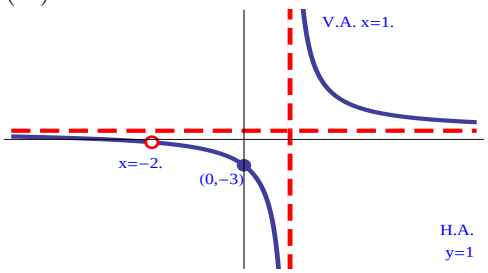
For which values of k does the system above have a unique solution?

(A) All $k \neq 0$ (B) All $k \neq 3$ (C) All $k \neq -3$ (D) All $k \neq 1$ (E) All $k \neq -1$

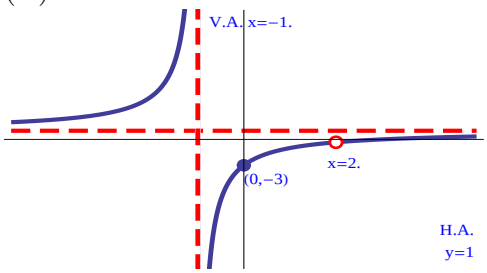
6. Which of the following could be the graph of $f(x) = \frac{x^2-5x+6}{x^2-x-2}$?

Hint: The function can be factored as $f(x) = \frac{(x-2)(x-3)}{(x+1)(x-2)}$

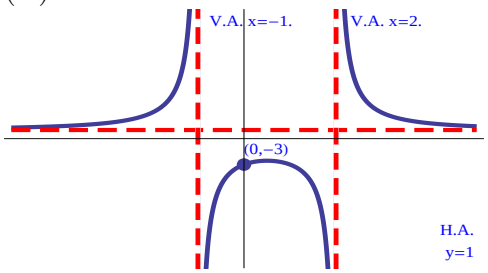
(A)



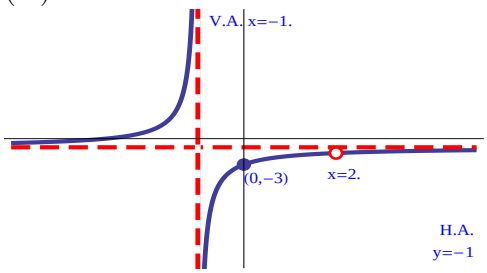
(B)



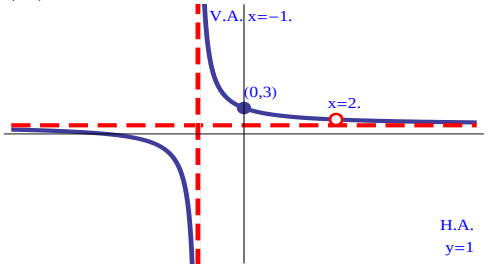
(C)



(D)



(E)



7. Calculate the slope of the tangent line to the graph of $2x^3 + 2y^3 = 9xy$ at the point $(2, 1)$.

   (A) 4/5 (B) 5/4 (C) $-2/5$ (D) $-5/4$ (E) 8

8. Let $f(x) = 6x^2 - 3$. Calculate the difference quotient of f. Recall that the difference quotient is given by $\frac{f(x+h)-f(x)}{h}$.

   (A) $12xh + 6h^2$ (B) $6(x + h) - 3$ (C) $12x$ (D) $12x + h$ (E) $12x + 6h$

9. A table of values for functions f and g, and their derivatives f' and g' is given. Calculate the derivative of $f(g(x))$ when $x = 1$.

x	f(x)	f'(x)	g(x)	g'(x)
-1	2	3	0	-3
0	1	1	1	1
1	-1	4	-1	-2
2	3	-1	3	-1

   (A) -8 (B) -4 (C) 3 (D) -6 (E) 6

10. The inverse of the matrix $C = \begin{pmatrix} -3 & 5 \\ 2 & -4 \end{pmatrix}$ is $C^{-1} = -\frac{1}{2}\begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix}$. Use this fact to solve the system: $CX = D$ where $D = \begin{pmatrix} 5 & 0 \\ -2 & 4 \end{pmatrix}$ and X is an unknown 2×2 matrix.

    (A) $\begin{pmatrix} 10 & 20 \\ 4 & 12 \end{pmatrix}$ (B) $\begin{pmatrix} -5 & -10 \\ -2 & -6 \end{pmatrix}$ (C) $\begin{pmatrix} -10 & -12.5 \\ 0 & -1 \end{pmatrix}$

    (D) $\begin{pmatrix} -25 & 20 \\ 18 & -16 \end{pmatrix}$ (E) $\begin{pmatrix} -25 & 20 \\ 14 & -26 \end{pmatrix}$

11. The reduced row echelon form for a linear system with variables x, y, z, and w in that order is given by $\left(\begin{array}{cccc|c} 1 & -1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -3 & 1 \end{array}\right)$. Which of the following represents the solution of the linear system?

(A) infinite number of solutions: $x = 0, y = 1, z = z, w = w$

(B) unique solution: $x = 0, y = 0, z = 1, w = 0$

(C) infinite number of solutions: $x = y - 2w, y = y, z = 1 + 3w, w = w$

(D) unique solution: $x = 1, y = -1, z = 0, w = 2$

(E) infinite number of solutions: $x = y - 2w, y = 1 + 3w, z = z, w = w$

12. A rectangle has sides of length l and w. Suppose that the sides of length l are increasing at 3 ft/sec and the sides of length w are decreasing at 2 ft/sec. How is the area of the rectangle changing when $l = 5$ and $w = 4$?

(A) increasing at 2 ft^2/sec (B) decreasing at 2 ft^2/sec

(C) decreasing at 1 ft^2/sec (D) decreasing at 6 ft^2/sec

(E) increasing at 20 ft^2/sec

13. Suppose that the TOTAL cost to produce x items is given by $C(x) = 3x^3 - 24x^2 + 55x$. Calculate the minimum AVERAGE cost.

(A) 6 (B) 42 (C) 7 (D) 0 (E) 4

14. The table below shows some values for a LINEAR function $f(x)$. Which of the following could be the value of a?

x	$f(x)$
2	1
4	a
a	16

(A) 0 (B) 1 (C) 4 (D) 5 (E) 7

15. Consider the equation: $y^3 - 2x^2y + x^2 + 2x = 8$. Find the intercepts.

(A) x-intercepts: $x = -4, 2$
y-intercepts: $y = -2, 2$

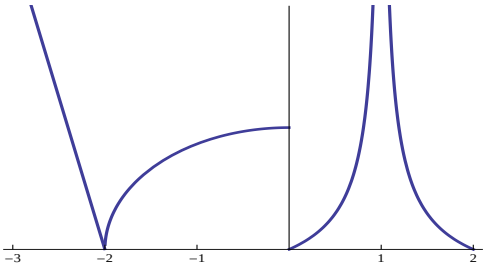
(B) x-intercepts: $x = 4, -2$
y-intercepts: $y = 2$

(C) x-intercepts: none
y-intercepts: $y = 2$

(D) x-intercepts: $x = -4, 2$
y-intercepts: $y = 2$

(E) x-intercepts: $x = -4, 2$
y-intercepts: none

16. A function $f(x)$ is graphed below. Determine where f is not continuous or not differentiable.



(A) not continuous at $x = 0$
not differentiable at $x = 0, 1$

(B) not continuous at $x = 1$
not differentiable at $x = 0, 1$

(C) not continuous at $x = 0, 1$
not differentiable at $x = 0, 1$

(D) not continuous at $x = 0, 1$
not differentiable at $x = -2, 0, 1$

(E) not continuous at $x = 0, 1$
differentiable everywhere

17. What is the center and radius of the circle: $x^2 + y^2 + x - 6y + \frac{29}{4} = 0$.

(A) center: $(1, -6)$
radius: $\frac{\sqrt{29}}{2}$

(B) center: $(-1, 6)$
radius: $\frac{\sqrt{29}}{2}$

(C) center: $(\frac{1}{2}, -3)$
radius: $\sqrt{2}$

(D) center: $(-\frac{1}{2}, 3)$
radius: $\sqrt{2}$

(E) center: $(-\frac{1}{2}, 3)$
radius: 2

18. $h(t) = -5t^2 + 20t + 25$ represents the height (in meters) above the ground of a ball thrown off the top of a 25 meter tall building t seconds after being thrown. How many seconds after throwing the ball will it hit the ground?

(A) $t = 2$s (B) $t = 4$s (C) $t = 5$s (D) $t = 0$s (E) $t = 45$s

19. Find all points on the graph of $f(x) = 3x^4 - 4x^3 + 5x$ where the corresponding tangent lines have slope 5.

(A) $(0, 0)$ (B) $(0, 0)$ and $(1, 4)$ (C) $(-1, 4)$

(D) $(0, 0)$ and $(-1, 4)$ (E) $(0, 5)$ and $(1, 5)$

20. Let $f(x) = \sqrt[3]{x-2}$. Calculate the equation of the tangent line when $x = -6$.

(A) $y - 2 = \frac{1}{12}(x - 6)$ (B) $y - 2 = -\frac{1}{12}(x - 6)$ (C) $y + 2 = -\frac{1}{6}(x + 6)$

(D) $y + 2 = -\frac{1}{12}(x + 6)$ (E) $y + 2 = \frac{1}{12}(x + 6)$

21. Let $g(t) = \frac{t^2}{t^2-2t+3}$. Calculate the instantaneous rate of change of g when $t = 2$.

(A) 4 (B) $\frac{4}{3}$ (C) $-\frac{4}{9}$ (D) $-\frac{4}{3}$ (E) $\frac{4}{9}$

22. Evaluate the limit:

$$\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x^2 - 4}$$

(A) $-\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $-\frac{1}{4}$ (D) 0 (E) 1

23. Suppose a demand equation is given by $p = \sqrt{110 - x}$. Calculate the marginal revenue when $x = 10$.

(A) \$1 (B) \$9.50 (C) \$10.50 (D) \$.05 (E) \$10

24. Evaluate the limit:

$$\lim_{x \to 1^-} \frac{1}{(1-x)^3}$$

(A) 1 (B) 0 (C) ∞ (D) $-\infty$ (E) does not exist

25. Let $f(x) = \frac{x+3}{x-3}$. Calculate the average rate of change of f on the interval $[5, 7]$.

(A) $-\frac{3}{4}$ (B) $\frac{3}{4}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{1}{4}$

MATH 2003
TEST F
Some of the questions on this part of the exam require a calculator.

For problems 26 and 27:

Let x = number of units and p = price of each unit.

Suppose the supply and demand functions are as follows:

$$\text{Supply } p = \sqrt{5x - 800}$$
$$\text{Demand } p = \sqrt{900 - 3x}$$

26. Determine the price at market equilibrium.
    (A) 160.00 (B) 212.50 (C) 300.00 (D) 16.20 (E) 262.50

27. Determine the maximum revenue.
    (A) 3442.89 (B) 3464.10 (C) 200 (D) 17.32 (E) 1741.86

28. For which value of c is the following function continuous when $x = 5$?

$$f(x) = \begin{cases} 2x & x \leq 5 \\ \frac{6}{\sqrt{x-c}} & x > 5 \end{cases}$$

(A) 0 (B) 5 (C) 4.64 (D) 5.36 (E) 3.56

29. Find the slope of the graph of $y = \frac{\sqrt{x+1}}{x+3}$ when $x = 2$.

(A) 0.98400 (B) -0.011547 (C) 0.34641 (D) 8.66025 (E) -2.88675

30. Evaluate the following limit:

$$\lim_{h\to 0} \frac{\sqrt{(x+h)^2-4} - \sqrt{x^2-4}}{h}$$

(A) 1 (B) $\frac{x}{\sqrt{x^2-4}}$ (C) x (D) $\frac{1}{2\sqrt{x^2-4}}$ (E) It is undefined.

For problems 31 and 32, suppose:

$$f(x) = \frac{3x^3 - 4x^2 + 3x - 2}{6 + 9x + 2x^2 - x^3}$$

31. Find the asymptotes of f.

(A) Horizontal: $y = \frac{1}{2}$
Vertical: $x = -1.37228, -1, 4.37228$

(B) Horizontal: $y = \frac{1}{2}$
Vertical: $x = 1$

(C) Horizontal: $y = -3$
Vertical: $x = -1.37228, -1, 4.37228$

(D) Horizontal: none
Vertical: $x = -1.37228, -1, 4.37228$

(E) Horizontal: $y = -3$
Vertical: $x = 1$

32. Find the points where f has a horizontal tangent line.

(A) $(-29.8765, -2.9679), (-1.1625, 82.7133)$ (B) $(-29.8765, -1.1625)$

(C) $(29.8765, -3.10962), (1.1625, .0451796)$ (D) There are no such points

(E) $(-29.8765, 0), (1.1625, 0)$

33. A certain taxi company charges a fixed price for the first mile, and \$1.30 for each additional mile. If it costs you \$14 to travel 9 miles, how much were you charged for the first mile?

(A) \$2.30 (B) \$1.56 (C) \$4.90 (D) \$1.00 (E) \$3.60

34. Solve the following system of equations:

$$\begin{cases} 3a - b - 8c = -3 \\ a + 2b + 2c + d = 1 \\ 7a + 7b + 4d = 2 \end{cases}$$

(A) $a = 2, b = -2, c = 1, d = 1$ (B) $a = 2c - \frac{1}{7}d, b = -2c - \frac{3}{7}d, c = c, d = d$

(C) $a = 1, b = -2, c = 1, d = 2$ (D) $a = 2c - \frac{1}{7}d, b = -2c - \frac{3}{7}d, c = c, d = 1$

(E) No solution

35. Let $A = \begin{pmatrix} 2 & 3 & -4 & 2 \\ 1 & -2 & 3 & -1 \\ -3 & 4 & -1 & 4 \\ 4 & -1 & 2 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$. Solve the equation $AX = C$.

(A) $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ (B) $\begin{pmatrix} 3/9 - k \\ 8/9 + 2k \\ k \\ 1/3 \end{pmatrix}$ (C) $\begin{pmatrix} 8 \\ 13 \\ 13 \\ 1 \end{pmatrix}$ (D) $\begin{pmatrix} 8/9 \\ 13/9 \\ 13/9 \\ 1/3 \end{pmatrix}$ (E) No solution.

Answers to Test F

1) B	2) D	3) E	4) E	5) C
6) B	7) B	8) E	9)D	10) B
11) C	12) A	13) C	14) E	15) D
16) D	17) D	18) C	19) B	20) E
21) E	22) A	23) B	24) C	25) A
26) D	27) B	28) C	29) B	30) B
31) C	32) A	33) E	34) E	35) D