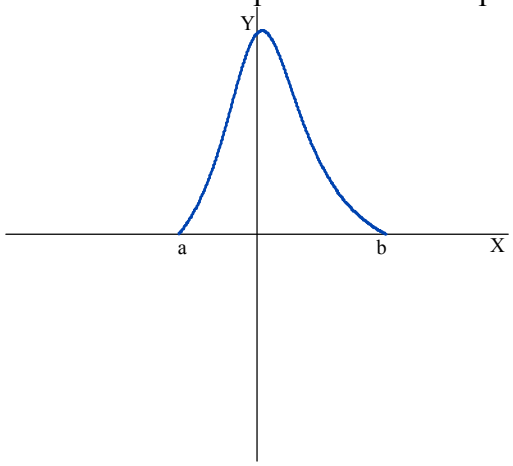
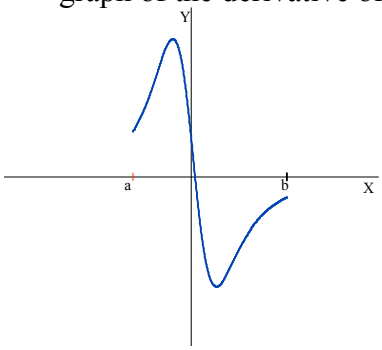
Math 2205
Additional problems for the Final Exam Manual
A calculator is NOT permitted on this part

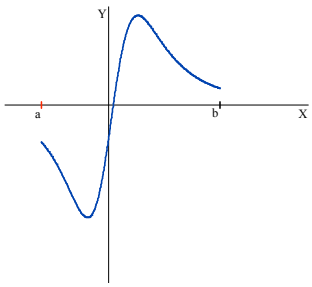


1. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

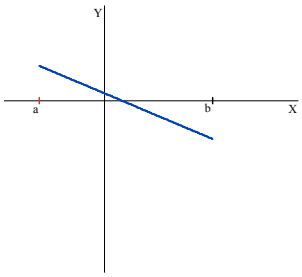
a)



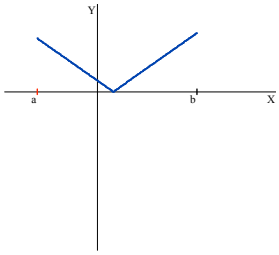
b)



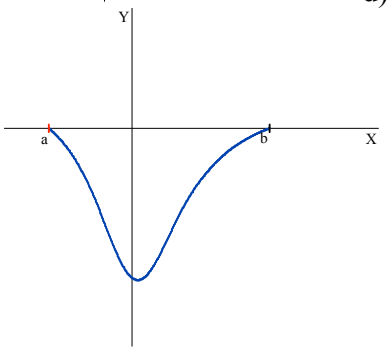
c)



d)



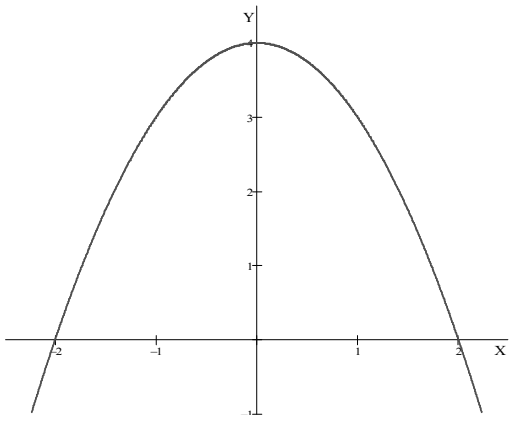
e)



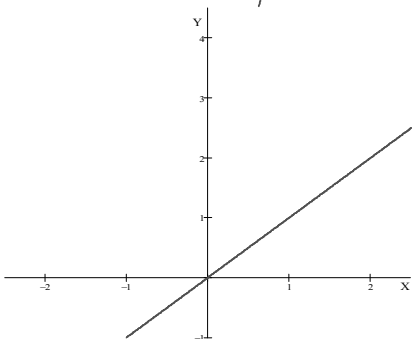
2. The graph of the derivative of *f*, f', is shown in the figure below.

This is the graph of f', the derivative of *f*.

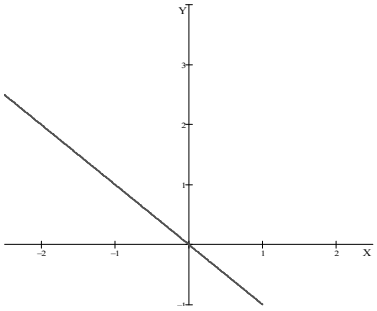
Which of the above is the graph of the function *f*?



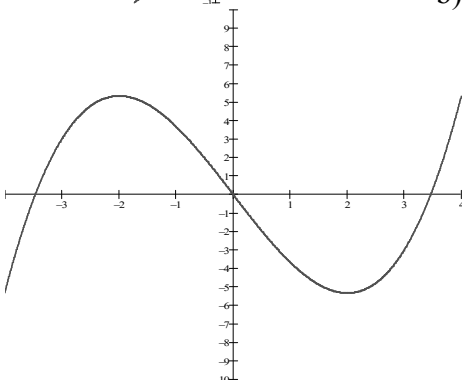
a)



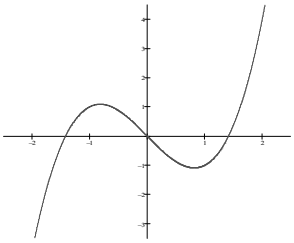
b)



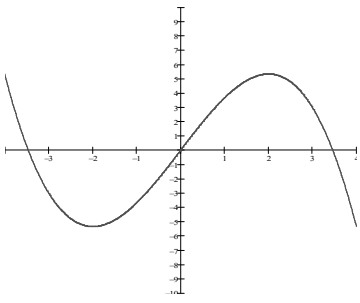
c)



d)



e)



3. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

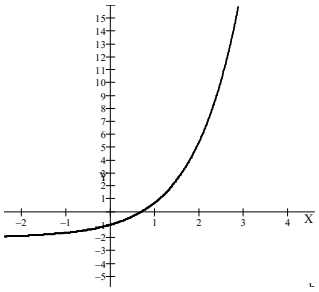
a) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
b) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
c) f has a relative minimum at $x = -2$ and at $x = 2$.
d) f has a relative maximum at $x = -2$ and at $x = 2$.
e) It cannot be determined if f has any relative extrema.

4. Let f be the function defined for all real numbers x such that $f'(x) = \dfrac{|x^2 - 4|}{x - 2}$.

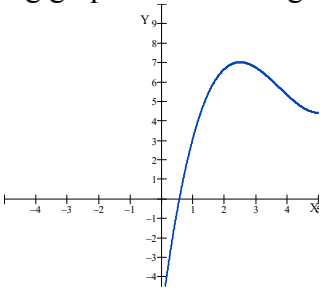
Then f is a decreasing function on the interval

a) $2 < x < \infty$
b) $-\infty < x < 2$
c) $0 < x < 2$
d) $-\infty < x < \infty$
e) $-2 < x < 2$

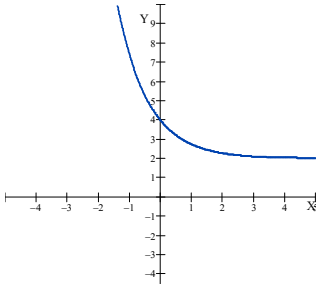
5. Which of the following graphs is increasing at a decreasing rate at $x = 2$?



$f(x)$



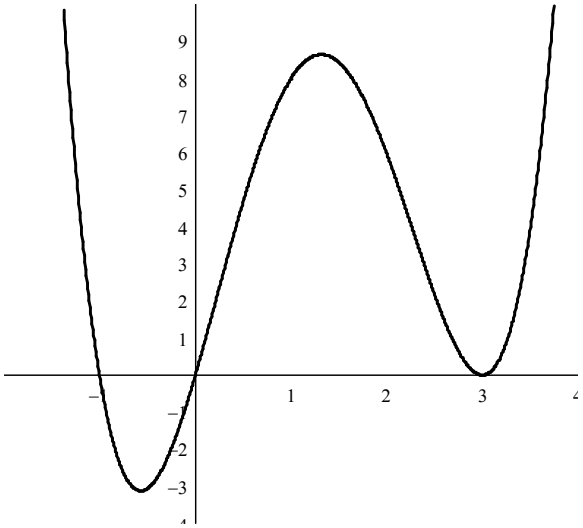
$g(x)$



$h(x)$

a. f only
b. g only
c. h only
d. f and g only
e. f and h only

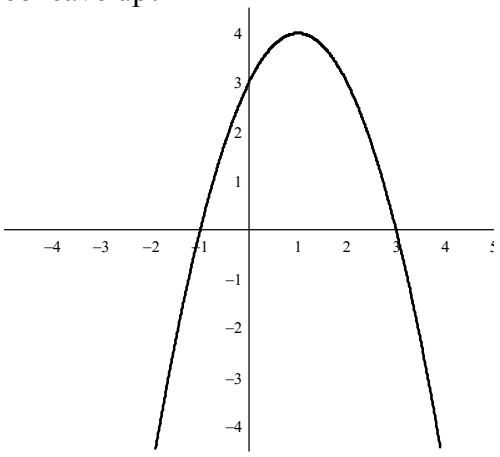
6. The second derivative of a function f, $f''(x) = x(x+1)(x-3)^2$, is given by the graph below:



For what values of x does f have a point of inflection?

a) $x = 0$ and $x = -1$ only
b) $x = 0$ and $x = 3$ only
c) $x = -1$ and $x = 3$ only
d) $x = -1$ and $x = 0$ and $x = 3$
e) f has no points of inflection

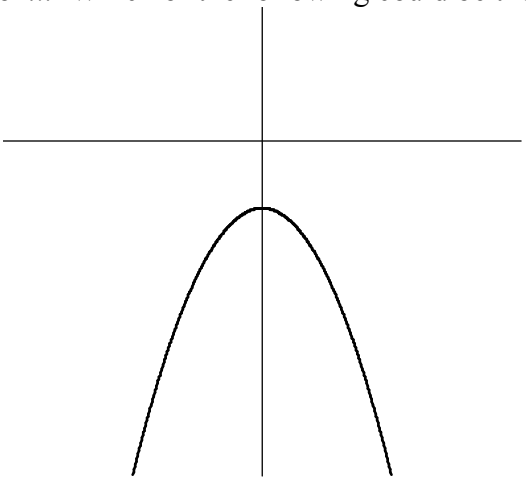
7. The graph of f'', the second derivative of the function f is shown below. On what intervals is f concave up?



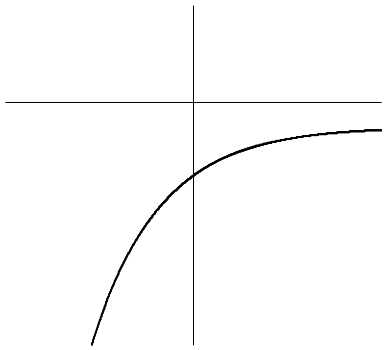
a) f is concave up for NO values of x
b) f is concave up for all values of x, $(-\infty, \infty)$
c) f is concave up on the intervals $(-\infty, -1)$ and $(3, \infty)$
d) f is concave up on the interval $(-1, 3)$
e) f is concave up on the interval $(-\infty, 1)$

8. The function f has the property that $f(x)$, $f'(x)$, and $f''(x)$ are negative for all values of x. Which of the following could be the graph of f?

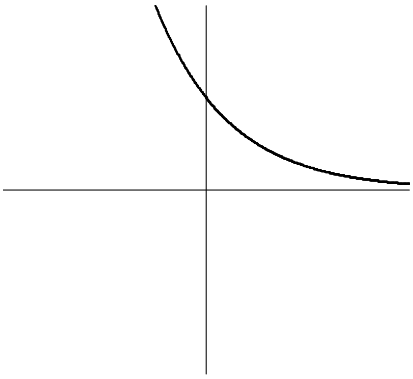
a)



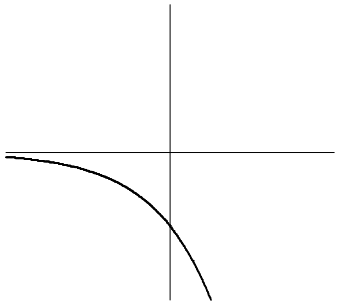
b)



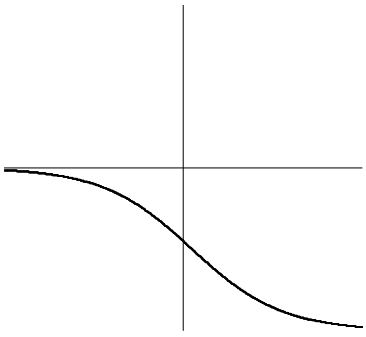
c)



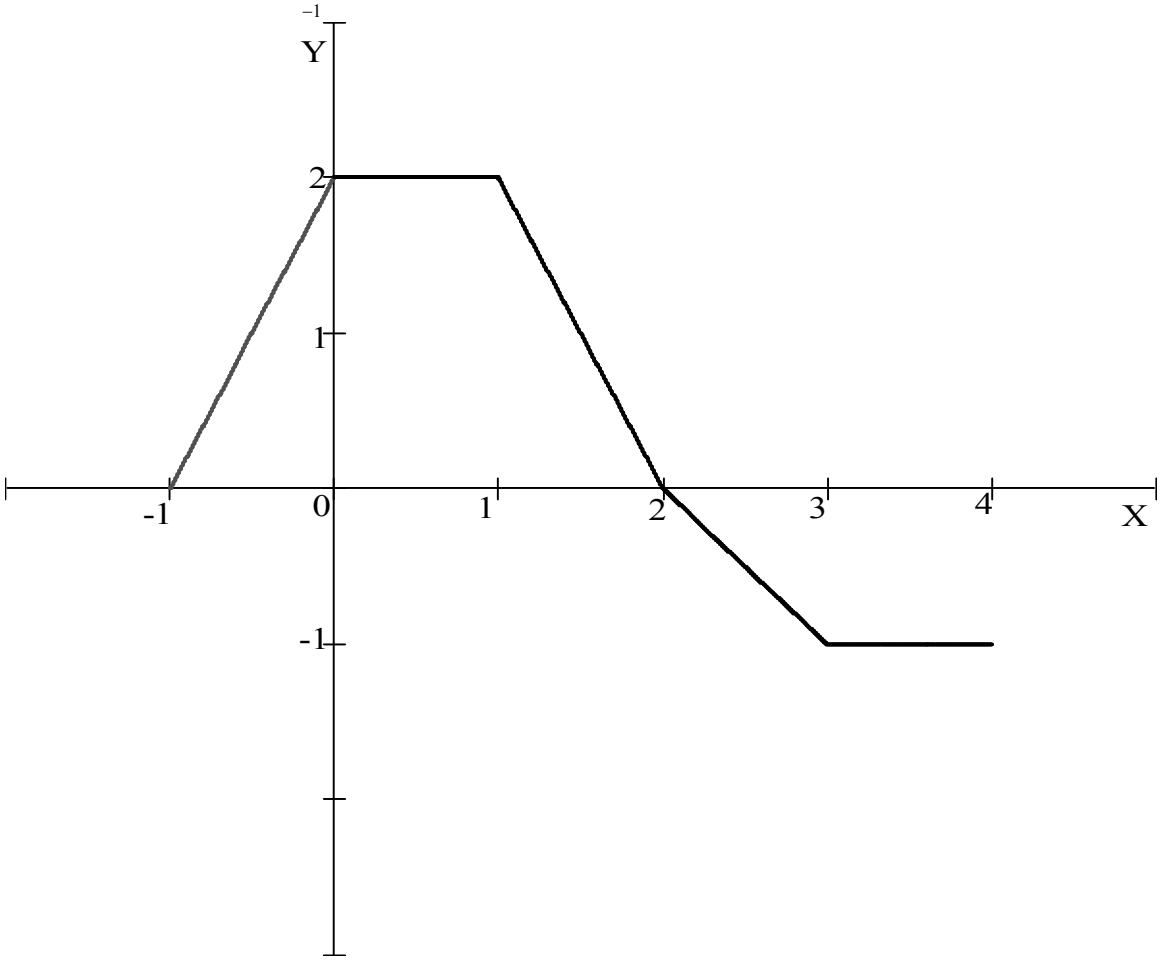
d)



e)



9. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown below. What is the value of $\int_{-1}^{4} f(x)\,dx$?



a) 1 b) 2.5 c) 4 d) 5.5 e) 8

10. If f is a linear function and $0 < a < b$, then $\int_{a}^{b} f''(x)\,dx =$

a. 0
b. 1
c. $\frac{ab}{2}$
d. $b - a$
e. $\frac{b^2 - a^2}{2}$

11. If $f(x)=\begin{cases}\ln x & \text{for } 0<x\le 2\\ x^2\ln 2 & \text{for } 2<x\le 4\end{cases}$, then $\lim_{x\to 2} f(x)$ is

a) $\ln 2$ b) $\ln 8$ c) $\ln 16$ d) 4 e) nonexistent

12. If $F(x)=\int_0^x \sqrt{t^3+1}\,dt$, then $F'(2)=$

a) -3 b) -2 c) 2 d) 3 e) 9

13. If $f''(x)=x(x+1)(x-2)^2$, then the graph of f has inflection points when $x=$

a) -1 only b) 2 only c) -1 and 0 only
d) -1 and 2 only e) -1, 0, and 2 only

14. What are all the values of k for which $\int_{-3}^{k} x^2\,dx=0$?

a) -3 b) 0
c) 3 d) -3 and 3 e) -3, 0, and 3

15. The function f is given by $f(x)=x^4+x^2-2$. On which of the following is f increasing?

a) $\left(-\frac{1}{\sqrt{2}},\ \infty\right)$
b) $\left(-\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right)$
c) $\left(-\infty,\ -\frac{1}{\sqrt{2}}\right)$
d) $(-\infty,\ 0)$
e) $(0,\ \infty)$

16. The function f is differentiable and has values as shown in the table below. Both f and f' are strictly increasing on the interval $0 \le x \le 5$. Which of the following could be the value of $f'(3)$?

x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.2	45	48.05

A) 20
B) 27.5
C) 29
D) 30
E) 48.05

17. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{3x^2}{y}$ with the initial condition $y(3) = -2$?

a) $y = 2e^{x^3}$
b) $y = 2e^{x^3-27}$
c) $y = x^3 - 25$
d) $y = \sqrt{2x^3 - 50}$
e) $y = -\sqrt{2x^3 - 50}$

18. Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(4) = 12$ and $g(5) = 18$. Of the following, which is a possible value for $g(6)$?

A) 15 B) 18 C) 21 D) 24 E) 27

19. The area of the region bounded by the graphs of $y = x^2$ and $y = \sqrt{x}$ is

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$ e) 1

20. If f has two continuous derivatives on $[5, 10]$, then $\int_5^{10} f''(x)\,dx =$

a) $f''(10) - f''(5)$
b) $f'(10) - f'(5)$
c) $f(10) - f(5)$
d) $f'(5) - f'(10)$
e) $f'''(10) - f'''(5)$

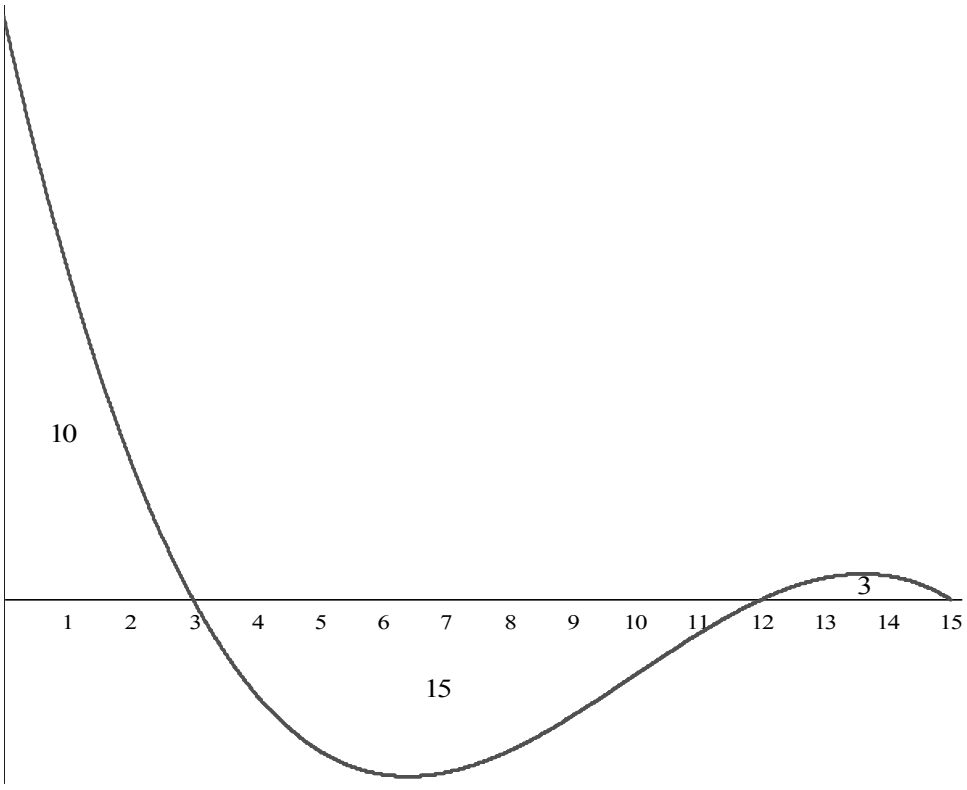
21. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

a) -21 b) -13 c) 0 d) 13 e) 21

22. Let *f* be a function with a first derivative given by $f'(x) = x^2(x-4)(x-7)$.
Then *f* has a relative maximum at $x =$

a) 0 only
b) 4 only
c) 7 only
d) 4 and 7 only
e) 0, 4, and 7

23. The graph of $y = g'(x)$ on the interval $0 \le x \le 15$ is shown below. The numbers in the regions give the area of the region. If $g(0) = 8$, what is $g(15)$?



a) -2 b) 2 c) 6 d) 28 e) 36

24. If $f(x)=2x$, $g(x)=x^2$, and $h(x)=2^x$, which of the following limits is equal to zero?

a) $\lim_{x\to\infty}\frac{g(x)}{f(x)}$ b) $\lim_{x\to\infty}\frac{g(x)}{h(x)}$ c) $\lim_{x\to\infty}\frac{h(x)}{f(x)}$ d) $\lim_{x\to\infty}\frac{h(x)}{g(x)}$

e) $\lim_{x\to\infty}\frac{h(x)}{f(x)g(x)}$

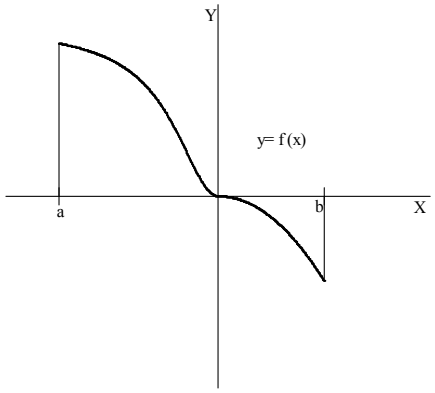
25. An object moves in a straight line with acceleration $a(t)=t^2\,\frac{ft}{\sec^2}$. If the initial velocity of the object is $-72\,\frac{ft}{\sec}$, at what time *t* does the particle change direction?

a) $t=\sqrt[3]{6}$
b) $t=6$
c) $t=6\sqrt{2}$
d) $t=36$
e) Never changes direction

26. Which of the following functions satisfies $0<f''(x)<f'(x)<f(x)$ for all values of *x*?

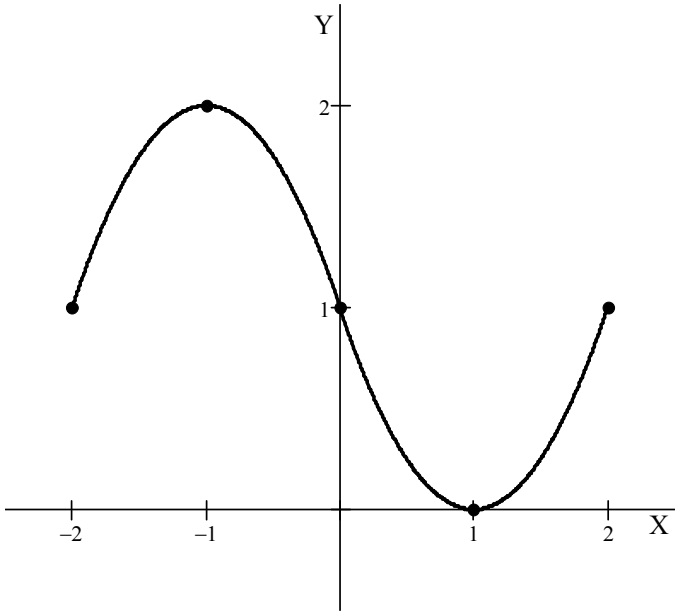
a) $f(x)=e^{-x}$
b) $f(x)=e^{\frac{x}{2}}$
c) $f(x)=e^{x}$
d) $f(x)=e^{2x}$
e) $f(x)=e^{x^2}$

27. Let f be a continuous function as shown in the graph below. The area of the region bounded by $f(x)$, the x-axis, and $x = a$ is 5. If $\int_a^b f(x)\,dx = 3$, then $\int_0^b f(x)\,dx$ is



a) -3 b) -2 c) 2 d) 8 e) cannot be determined

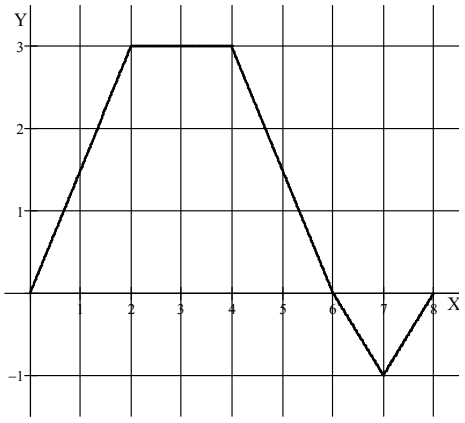
28. Given the graph of $y = f(x)$ shown below. Which of the following values is largest?



a) $f(0)$ b) $f'(0)$ c) $\lim_{h\to 0}\dfrac{f(h)-1}{h}$

d) $\dfrac{f(1)-f(-1)}{2}$ e) $\dfrac{f'(1)-f'(-1)}{2}$

Questions 29 and 30 refer to the graph below.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

29. At what time t does the bug change direction?

a) 2 b) 4 c) 6 d) 7 e) 8

30. What is the total distance traveled by the bug from $t = 0$ to $t = 8$?

a) 14 b) 13 c) 11 d) 8 e) 6

31. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

a) $2e^{kty}$ b) $2e^{kt}$ c) $e^{kt} + 3$ d) $kty + 5$ e) $\frac{1}{2}ky^2 + \frac{1}{2}$

A Calculator is May be Used for the Following Problems

32. The function f is continuous on the closed interval $2 \le x \le 8$ and has values as given in the table below. Using the subintervals $[2,5]$, $[5,7]$ and $[7,8]$, approximate $\int_2^8 f(x)\,dx$ using a left-hand approximation with three rectangles.

x	2	5	7	8
$f(x)$	10	30	40	20

a) 110 b) 130 c) 160 d) 190 e) 210

33. Population N grows according to the equation $\frac{dN}{dt} = kN$, where k is a constant and t is measured in years. If the population doubles every 10 years, then $k =$

a) 0.069 b) 0.200 c) 0.301 d) 3.322 e) 5.000

34. The rate at which water is sprayed onto a field of vegetables is given by $R(t) = 2\sqrt{1+5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \le t \le 4$, what is the average rate of water flow, in gallons per minute?

a) 8.458 b) 13.395 c) 14.691 d) 18.916 e) 35.833

35. Suppose that a five-year rate of projection growth of population trends suggests that t years from now, the population of a certain community will be P thousand, where

$$P(t) = -t^3 + 9t^2 + 48t + 200$$

At what time will the **RATE** of population growth be maximized?

a) $t = 3 years$
b) $t = 13 years$
c) $t = 15 years$
d) $t = 60 years$
e) $t = 75 years$

36. It is projected that t months from now, the average price per unit for goods in a certain sector of the economy will be P dollars where

$$P(t) = -t^3 + 7t^2 + 200t + 300$$

At what **rate** will **the <u>rate of price increase</u>** be changing with respect to the time 5 months from now?

b) \$195 unit per month
b) –\$16 unit per month per month
c) \$2.33 unit per month per month
d) \$10.83 unit per month per month
e) –\$10 unit per month per month

37. An object traveling in a straight line has position $s(t)$ at time t. If the initial position is $s(0) = 3$ and the velocity of the object is $v(t) = \sqrt[3]{1+2t^2}$, what is the position of the object at time $t = 2$?

a) 7.312
b) 5.933
c) 2.933
d) 8.312
e) 24

38. The temperature of a cup of hot chocolate changes at the rate of $r(t) = -6e^{-0.12t}{}^{\circ}C$ per minute with t measured in minutes. Calculate the change in the temperature from $t = 0$ to $t = 3$ minutes.

a) decreases $15.116^{\circ}C$
b) decreases $1.814^{\circ}C$
c) decreases $0.502^{\circ}C$
d) decreases $0.720^{\circ}C$
e) decreases $10.720^{\circ}C$

39. If $f(x) = x^3 + x$ and $g(x)$ is the inverse function of $f(x)$, then $g'(1) =$

a) –0.5
b) 0.003
c) 0.077
d) 0.25
e) 0.417

40. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}$$

$E(t)$ is measured in people per hour and time t is measured in hours after midnight. The function is valid for $9 \le t \le 23$, the hours during which the park is open. How many people enter the park from 9 AM to 5 PM ($t = 17$)?

a) 975 b) 6,004 c) 7,276 d) 8,366 e) 9,637

41. Rank the following functions from <u>slowest growing to fastest growing</u> in that order: e^x, $\ln x$, 5^x, x, 2^x

a) e^x, 5^x, $\ln x$, x, 2^x
b) 2^x, e^x, 5^x, $\ln x$, x
c) $\ln x$, x, 2^x, e^x, 5^x
d) x, $\ln x$, 2^x, e^x, 5^x
e) x, $\ln x$, 5^x, 2^x, e^x

42. What nominal interest rate has an effective annual yield of 5% under continuous compounding?

a) 4.88 %
b) 5.10 %
c) 5.127 %
d) 5.21 %
e) 5.217 %

43. $\frac{d}{dx}\left(xe^{\ln x^2}\right) =$

a) x^3
b) $3x^2$
c) $2x$
d) x^2
e) $e^{\ln x^2}$

44. The function f *is* twice differentiable with $f(2)=1,\ f'(2)=4,$ and $f''(2)=3$. What is the value of the approximation of $f(1.9)$ using the tangent to the graph of f at $x=2$?

a) 0.4
b) 0.6
c) 0.7
d) 1.3
e) 1.4

45. Consider the curve defined by $-8x^2+5xy+y^3=-149$. Write an equation for the line tangent to f at the point $(4,-1)$ and using this line approximate $f(4.2)$.

a) -0.4
b) 0.4
c) -0.373
d) 0.373
e) -0.6

f) ANSWERS

Problem	Answer	Section in Text
1	A	3.2
2	E	3.2
3	B	3.1
4	B	3.2
5	B	3.3
6	A	3.3
7	D	3.3
8	D	3.3
9	B	5.6
10	A	5.6
11	E	4.5
12	D	5.7
13	C	3.3
14	A	5.6
15	E	3.2
16	D	3.2
17	E	5.2
18	E	3.3
19	B	5.8
20	B	5.6
21	B	5.6
22	B	3.1
23	C	5.6
24	B	4.2
25	B	5.2
26	B	4.4
27	B	5.6
28	A	3.1
29	C	3.2
30	B	5.7
31	B	4.7 OR 5.2
32	B	5.4
33	A	4.7
34	C	5.7
35	A	3.5
36	B	3.2
37	B	5.2
38	A	5.8
39	E	4.1
40	B	5.8
41	C	4.5
42	C	4.7
43	B	4.5
44	B	3.6
45	A	3.6

Section in Text	Problem
3.1	3, 22, 28
3.2	1, 2, 4, 15, 16, 29, 36
3.3	5, 6, 7, 8, 13, 18
3.4	
3.5	35
3.6	44, 45
4.1	39
4.2	24
4.3	
4.4	26
4.5	11, 41, 43
4.6	
4.7	31, 33, 42
5.1	
5.2	17, 25, 31, 37
5.3	
5.4	32
5.5	
5.6	9, 10, 14, 20, 21, 23, 27
5.7	12, 30, 34
5.8	19, 38, 40