

BARUCH COLLEGE
USING THE TI-89 IN
PRECALCULUS AND CALCULUS

by

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PRELIMINARIES

On the upper left side of the calculator there are three keys labeled 2nd (yellow), \blacklozenge (green) and alpha (purple). These keys are used to access the commands that appear above the other keys. Thus, in order to access the function e^x , you would first press the green \blacklozenge key and then press x since the function e^x appears in green above the x key.

The backspace key \leftarrow to the left of the CLEAR key is used to delete the previous character entered. The four blue cursor keys that appear above the CLEAR key are used to move in one of the directions indicated.

Press ON. The Home screen shown in Figure 1 appears. The toolbar appears at the top with the six menus shown. The blank area that appears next is the history area. The single line between the two horizontal lines is the command line (entry line). The blinking vertical line at the left end of the command line is the cursor. At the bottom the status line appears. More will be said about some entries that appear there later.

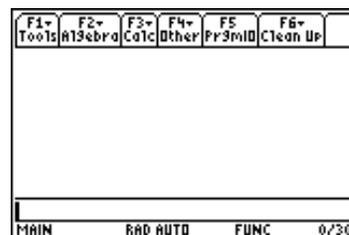


Figure 1

CLEARING THE HISTORY AREA AND COMMAND LINE

When a new problem is being worked on, it is useful to start with a blank screen like the one shown previously. Press 7 several times and then press ENTER several times so that characters appear in the history area and on the command line. Now press the blue F1 key to obtain Figure 2. Notice that the calculator looks as if it is defective since the first and third menu entries are fuzzy. Those menu entries are fuzzy simply because there is nothing they can currently be used for. When they can be used, they will become clear. Now press 8 (or press the down cursor until 8↓Clear Home is highlighted and press the blue ENTER key). The history area will now be clear but the command line will still have the 7s appearing on it. Press the CLEAR key to clear the command line. The screen now looks like Figure 1.



Figure 2

RESTORING THE CALCULATOR DEFAULTS

It is very desirable to make sure that your calculator operates the same way that the instructor's calculator does. Therefore, you should do the following as soon as possible unless you have a brand new calculator that you have not done anything with yet.

Press 2nd, 6 (yellow MEM). Figure 3 is the result.



Figure 3

Press F1 (reset).

Figure 4 appears (See the note below if a different screen appears).

Press the right cursor (in order to make the submenu for choice 1:RAM appear).



Figure 4

Figure 5 appears.

(Observe that whenever \blacktriangleright appears next to a menu item there is a submenu that is obtained by pressing the right cursor arrow.)

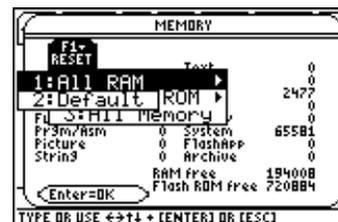


Figure 5

Now press 2 to select "Default" (or press the down cursor to highlight 2:Default and then press ENTER).

Figure 6 appears.

Pressing ENTER brings up a screen (not shown) that requires ENTER to be pressed a second time.



Figure 6

Note: Calculators with an older operating system might have Figures 4 and 5 replaced by a single screen where choice 3:Default is the proper selection.

After the calculator defaults have been reset, YOU SHOULD NOT TOUCH THE MODE KEY FOR THE REST OF THE SEMESTER.

EXACT, APPROXIMATE AND AUTO CALCULATIONS

If the command $1/6+5/9$ ENTER is executed, Figure 7 results.

The default mode for performing calculations is AUTO. This should never be changed. This mode means that the calculator will first try to find an exact answer. However, if an exact answer cannot be found within a certain amount of time, the calculator will then try to find an approximate (decimal) answer. It will often happen that you will want the approximate answer even when the calculator can find the exact answer. For example, if you want to know what π^3 equals as a decimal, it is not helpful if the calculator says it equals π^3 .

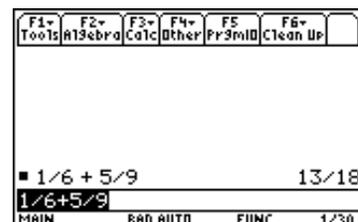


Figure 7

There are two ways to force the calculator to find the decimal (approximate) result. They are as follows.

Entering a decimal point for any number on the command line produces a decimal result.

If the green \blacklozenge key is pressed before pressing ENTER, a decimal result occurs. Thus, if \blacklozenge ENTER is pressed for Figure 7, then Figure 8 results. That is what the green \approx above ENTER means.

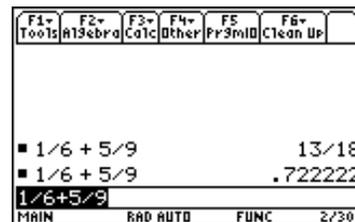


Figure 8

NUMERICAL ACCURACY (AND USING PREVIOUS HISTORY AREA ITEMS)

By default, numbers are displayed to six digit accuracy. However, they are stored internally to an accuracy of 12 digits. If you ever want to see (or use) a decimal result, that can be done. Any item in the history area can be recovered as follows. Press the up cursor until the item is highlighted. If the up cursor is pressed for Figure 8 then Figure 9 results. Pressing ENTER then produces Figure 10.

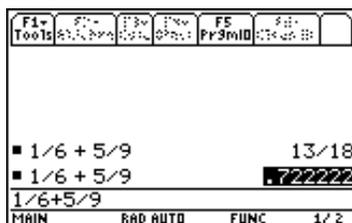


Figure 9

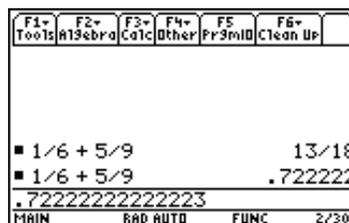


Figure 10

The “2/30” that appears on the right side of the bottom status bar of Figure 10 indicates that the history area has 2 lines and the last 30 lines entered will be saved. That means even entries in the history area that have disappeared off of the top of the screen can be recovered by using the up cursor arrow as long as the entry was on one of the previous 30 lines.

EVALUATING EXPRESSIONS

The key to the left of 7 with the vertical line on it is the “when” or “such that” key. The expression $7x^2 - 5x + 2$ can be evaluated “when” $x = 3.7$ by entering the following on the command line:
 $7x^2-5x+2|x=3.7$ ENTER

Figure 11 shows the result is 79.33.

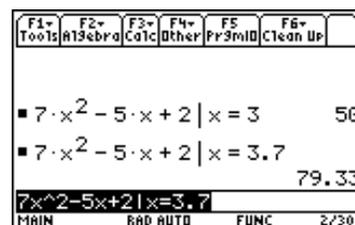


Figure 11

EDITING THE COMMAND LINE

When a command is executed, the command line is highlighted as in Screen 9. If a key (other than the left or right cursor arrow) is pressed, the command line is cleared and the keystroke made becomes the first character on the command line. In order to edit the command line, proceed as follows: Press either the left or right cursor arrow. The command line will cease being highlighted. If the right cursor arrow is pressed, the blinking cursor will be placed at the right end of the command. If the left cursor arrow is pressed, the blinking cursor will be placed

at the beginning of the command line. The left and right cursor arrows can now be used to move the cursor to any location on the command line. At any given location the character to the left of the cursor can be deleted by pressing the backspace key (\leftarrow). The character to the right of the cursor can be deleted by pressing $\blacklozenge \leftarrow$ (del = delete).

BASIC ALGEBRA

The calculator will perform some algebraic operations without the user having to provide any special commands. For example, Figure 12 shows what happens when the sum of three algebraic fractions is entered on the command line.

Press $2/x + 2/(1-x) + 4/(x^2-1)$ ENTER

Note that if the numerator or denominator of a fraction consists of more than a single number or variable, then you should enclose it within parentheses.

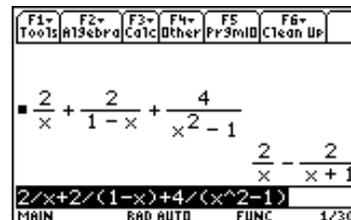


Figure 12

Pressing the blue F2 key in the Home screen produces the menu shown in Figure 13. Note that the down arrow next to item 8 indicates the fact that the menu contains more items than those shown. Pressing the down cursor repeatedly would reveal “9:Trig” and “A:Complex” (both with submenus).

Press the blue ESC key to cancel the menu and return to Figure 12.



Figure 13

COMBINING ALGEBRAIC FRACTIONS: comDenom(expression)

Example: Simplify $\frac{2}{x} + \frac{2}{1-x} + \frac{4}{x^2-1}$

Solution:

Press the left cursor arrow for Figure 12 to place the cursor at the beginning of the command line and remove the highlighting. Then press F2 (Algebra) and select choice 6:comDenom. The screen now looks like Figure 14. The “...” on the command line indicates part of the command is off the screen and the right cursor arrow must be used in order to see it (if desired).

Press ENTER. The message “Missing)” appears because each function begins with an open parenthesis but the user must place the closing parenthesis at the end of the arguments. Press ESC so that the message disappears. Note that the cursor has been placed at the position where the calculator expects the closing parenthesis to appear.

Press)ENTER. Figure 15 appears.

The calculator combines the fractions and reduces the result.

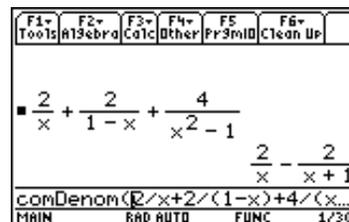


Figure 14

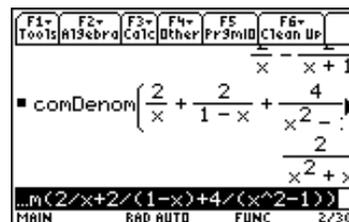


Figure 15

Notice the \square symbol in the history area. This indicates the fact that part of the expression is off the screen.

Answer:
$$\frac{2}{x} + \frac{2}{1-x} + \frac{4}{x^2-1} = \frac{2}{x^2+x}$$

Press the up cursor arrow twice to highlight the line and then press the right cursor arrow until you can see the end of the line as in Figure 16.

Press any digit. Nothing happens because you are not on the command line.

Press the down cursor arrow twice to return to the command line.

Clear the Home screen (F1, 8, CLEAR).

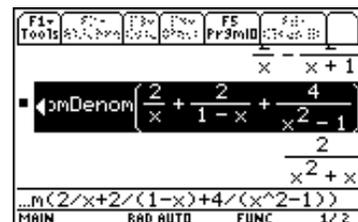


Figure 16

SOLVING EQUATIONS: solve(equation,variable)

Example: Solve $6x^4 + 7x^3 + x^2 + 7x - 5 = 0$

Solution: Press F2(Algebra) and ENTER (for choice 1:solve).

Press $6x^4+7x^3+x^2+7x-5=0,x$ ENTER

Figure 17 shows the result.

The calculator has found the two exact real solutions to the equation $6x^4 + 7x^3 + x^2 + 7x - 5 = 0$: $x = 1/2$ and $-5/3$.

Notice that just entering the equation is not enough. You must have a comma followed by the variable to be solved for after it.



Figure 17

Example: Solve $3x^2 + 2xy - 5y^2 = 0$ for y.

Solution: Press F2(Algebra) and ENTER (for choice 1: solve).

Press $3x^2+2x*y-5y^2=0,y$ ENTER

Figure 18 shows the result.

The solutions are $y = -3x/5$ and $y = x$.

IMPORTANT: The calculator understands 5x means

5 times x. But since variables can have more than one letter, you must enter the times symbol between x and y if you want to multiply them. I.e. $x*y$ must be used.

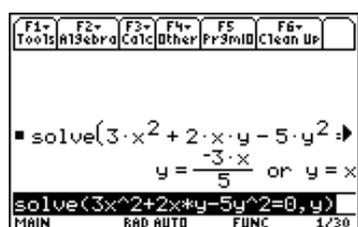


Figure 18

MULTIPLYING POLYNOMIALS: expand(expression)

Example: Multiply $(3x + 5)(2x - 1)(x + 1)$

Solution: Press F2 (Algebra) and select choice 3:expand

Press $(3x+5)*(2x-1)*(x+1)$ ENTER

The result is Figure 19.

Answer: $(3x + 5)(2x - 1)(x + 1) = 6x^3 + 13x^2 + 2x - 5.$

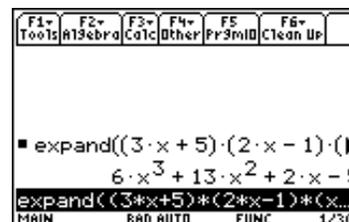


Figure 19

FACTORING: factor(expression)

Example: Factor $2x^3 + x^2 - 8x - 4$

Solution: Press F2 (Algebra) and select choice 2:factor.

Press $2x^3+x^2-8x-4$ ENTER

The result is Figure 20.

Answer: $2x^3 + x^2 - 8x - 4 = (x - 2)(x + 2)(2x + 1).$

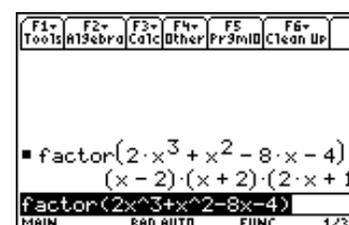


Figure 20

GRAPHING FUNCTIONS

GRAPHING PARABOLAS: $y = ax^2 + bx + c$

Example: Graph $y = x^2 - 4x - 3$ and find the turning point.

Solution: Functions to be graphed are entered by using the y= editor. Notice above the blue F1 key y= appears in green. Press the green \blacklozenge key and then the F1 key.

The result is Figure 21.



Figure 21

Press x^2-4x-3 ENTER

As soon as you press the first x the equation starts to appear on the command (entry) line. It does not move up to the top until you press ENTER. Your screen should now look like Figure 22.

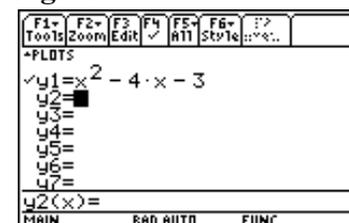


Figure 22

Press F2 (Zoom).

The result is Figure 23.

Press 6 (for ZoomStd)

(or cursor down to highlight 6:ZoomStd and press ENTER).



Figure 23

The result is Figure 24, the graph of $y = x^2 - 4x - 3$.

The axes are not labeled. It is assumed that you know that for ZoomStd the x-axis goes from -10 to 10 and the y-axis does the same. The tick marks for the x-axis are -10,-9,-8,-7,.....,8,9,10. The same is true for the y-axis. Notice that the turning point looks like it occurs at (2, -7). Trace can be used to get a good approximation as to where the point is located. F5 (Math) can be used to find it exactly.

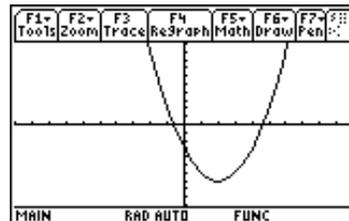


Figure 24

Press F3 (Trace).

The result is Figure 25. The coordinates of the point on the parabola near the (horizontal) middle of the screen are displayed and the cursor is blinking at that point. So (0.253, -3.949) is a point on the graph. As you press the right cursor, the values of y decrease until you get to (2.0253165, -6.999359). The point just before that was (1.7721519, -6.948085) and the point after that is (2.278481, -6.922448). Based on these facts you might estimate the turning point to be (2.025, -6.999). But all you can really say for sure is that it occurs at a value of x between 1.77 and 2.28 and that the value of y is less than -6.999.

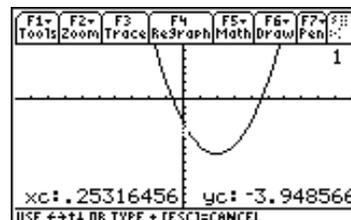


Figure 25

Press F5 (Math).

The result is Figure 26.

The point we are looking for is a low point on the graph, a minimum.

Press 3 (for Minimum).

The result is Figure 27.

The question "Lower Bound?" means you have to indicate a value of x to the left of the minimum. This can be entered numerically, but usually it is done by simply using the left or right cursor arrow to move the cursor to a point to the left of the minimum but reasonably nearby. Then press ENTER.



Figure 26

The result is Figure 28 (or close to it).

The new question "Upper Bound?" means you have to indicate a value of x to the right of the minimum.

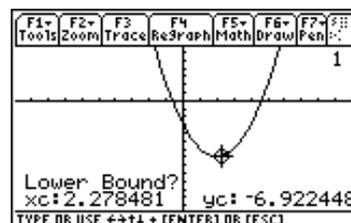


Figure 27

Use the right cursor arrow to move the cursor to the right of the minimum but reasonably nearby such as the point shown in Figure 29.

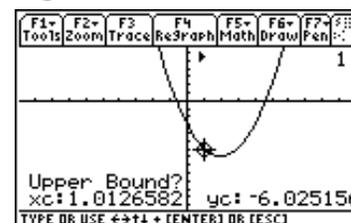


Figure 28

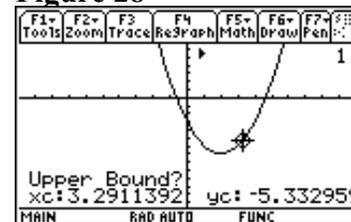


Figure 29

Press ENTER.

The result is Figure 30.

The turning point (minimum) is (2, -7).

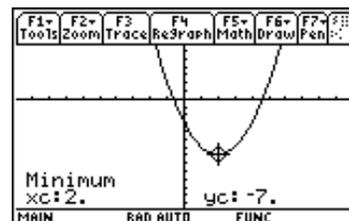


Figure 30

Example: Graph $y = x^2 - 12x + 9$ and find the turning point.

Solution: Return to the y= editor (green \blacklozenge , F1).

Move the cursor arrow to highlight y1 from the previous example.

Press CLEAR.

Press $x^2 - 12x + 9$ ENTER.

The result is Figure 31.

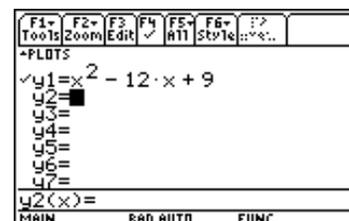


Figure 31

Press F2 (Zoom) and then 6 (ZoomStd).

The result is Figure 32.

The graph looks almost like a straight line, but you know that cannot be correct since the graph should be a parabola opening upward since the coefficient of x^2 is positive (1). So you should suspect the turning point is somewhere to the right of 2. You would like to see some of the y values for values of x to the right of $x = 2$.

Press F3 (Trace) and then press the right cursor arrow to view values of y for values of x between 0 and 10. Even though the cursor disappears off of the bottom of the screen, note that you can still see the y values for the various x values. Do not go beyond $x = 10$. The values of y go down to close to -27 (near $x = 6$) and then go up. So the turning point should be near (6, -27). It would be good to get the point on the graph and confirm this. So it would be good to graph the parabola for values of x between -5 and 15 (with tick marks one unit apart) and values of y between -30 and 20 (with tick marks 5 units apart in order to keep them from getting too close together). The exact end points chosen are not important. All that is really important at this time is that the point (6, -27) should appear on the graph. Notice that above the F2 key the word "WINDOW" appears in green. The end points and tick mark separation can be specified in the WINDOW screen.

Press green \blacklozenge F2 (WINDOW).

The result is Figure 33.

Press -5 ENTER

(Use the grey negative (-) key not the black subtraction key!)

Press 15 ENTER

Press 1 ENTER (or just cursor down to ymin)

Press -30 ENTER (always grey negative (-) for a negative number)

Press 20 ENTER

Press 5 ENTER

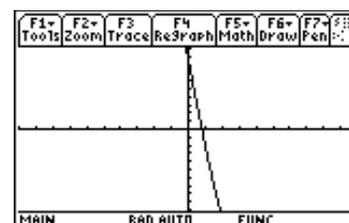


Figure 32

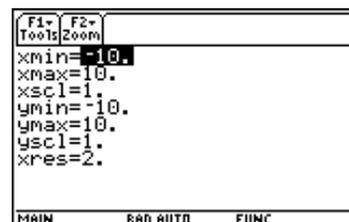


Figure 33

The result is Figure 34.

This means that the x values go from $x_{\min}=-5$ to $x_{\max}=15$ with tick marks $x_{\text{scl}}=1$ unit apart. The y values go from $y_{\min}=-30$ to $y_{\max}=20$ with tick marks $y_{\text{scl}}=5$ units apart.

You can ignore $x_{\text{res}}=2$ (the resolution). It can have values of 1, 2, 3 or 4. It is okay to leave it at 2 all of the time.

Above the F3 the word "GRAPH" appears in green. In order to see the graph with the window values specified, we need to return to the GRAPH screen.

Press green \blacklozenge , F3 (GRAPH).

The result is Figure 35.

The x-axis tick marks correspond to -5,-4,-3,-2,-1,0,1,...,13,14,15

The y-axis tick marks correspond to -30,-25,-20,-15,...,10,15,20

Now find the turning point as before:

Press F5 (Math) then choice 3 (Minimum)

Make sure the cursor is to the left of the turning point and then press ENTER for the Lower Bound.

Move the cursor to the right of the turning point and then press ENTER for the Upper Bound.

The result is Figure 36.

The turning point is (6, -27) exactly.

Example: Graph $y = -x^2 + 31x - 240$ and find the turning point.

Solution: Go to the y= editor, clear y1, and enter the new function. Press F2 (Zoom) and then ZoomStd. After seeing the word "busy" on the bottom of the screen for a little while, you see a blank graph. This is not surprising since $y = -240$ at $x = 0$ and the graph only shows y values between -10 and 10. The parabola opens downward since the coefficient of x^2 is negative (-1). Press F3 (Trace) followed by many presses for the right cursor arrow to see various y values. Notice they are increasing (going from -236.092 to -35.82599 when you get to $x = 9.4936709$). So we are on the side of the parabola that is going up and the turning point should be somewhere to the right of $x = 10$ at which point the parabola would go down. This suggests a new graphing window where x goes from 0 to 20 (with tick marks one unit apart) and y goes from -200 to 100 (with tick marks 25 units apart).

Enter the following graphing window (see last example):

$x_{\min}=0$, $x_{\max}=20$, $x_{\text{scl}}=1$, $y_{\min}=-200$, $y_{\max} = 100$, $y_{\text{scl}}=25$.

The result is Figure 37.

This can be used as is, or we could obtain a graph that looks a little nicer by concentrating on the turning point that appears to be near the point (15, 0).



Figure 34

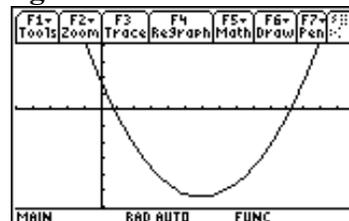


Figure 35

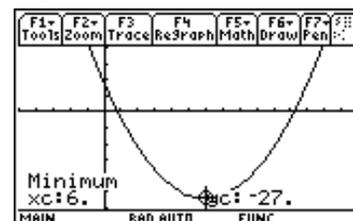


Figure 36

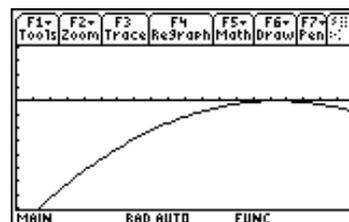


Figure 37

Entering the following graphing window

$x_{\min}=5$, $x_{\max}=25$, $x_{\text{scl}}=1$, $y_{\min}=-100$, $y_{\max}=25$, $y_{\text{scl}}=25$.

we obtain Figure 38. Notice there are tick marks on the vertical left side but no axis because the y-axis is off the screen to the left.

This graph has a maximum, not a minimum.

Press F5 (Math) and select choice 4 (Maximum). Then proceed to select the Lower and Upper Bounds as before.

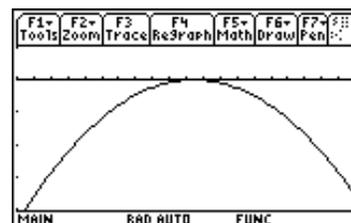


Figure 38

Figure 39 is the result.

The turning point indicated is (15.499997, 0.25).

(The exact turning point is (15.5, 0.25). Because the calculator uses numerical methods to find its answers, it sometimes stops slightly short of the precise answer.)

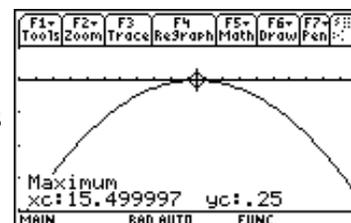
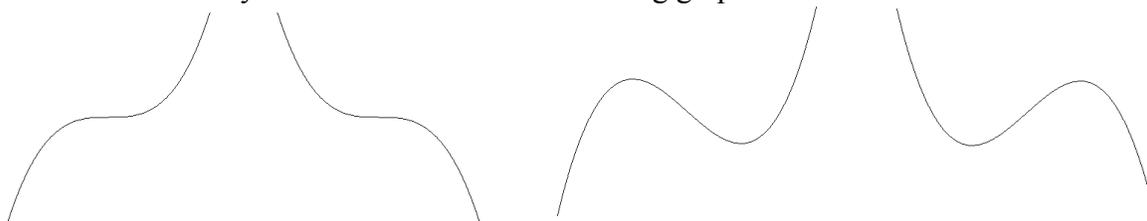


Figure 39

BASIC IDEA: IN ORDER TO GRAPH A FUNCTION WITH THE TI-89, YOU MUST FIRST DETERMINE THE IMPORTANT FEATURES OF THE FUNCTION THAT SHOULD APPEAR ON THE GRAPH. THEN MAKE SURE THE GRAPHING WINDOW SHOWS THOSE FEATURES. For example, for a parabola the important feature is the turning point.

GRAPHING CUBICS: $y = ax^3 + bx^2 + cx + d$.

A cubic function always looks like one of the following graphs.



Therefore, a graph of a cubic should show either the “squiggle” that appears in one of the first two graphs or two turning points.

Example: Graph $y = x^3 + 20x^2 + 75x$. Find the turning points (if any). Also find the zeros (values of x at the horizontal intercepts, the solutions to $0 = x^3 + 20x^2 + 75x$).

Solution: Figure 40 shows the graph using ZoomStd.

Activating Trace (F3) indicates the curve is rapidly rising near $x=0$, going over 1000 by the time you get to $x=6$. Using Trace too the left of $x=0$ the curve drops to close to $y = -79$ before coming back near $x=-5$. This suggests that y values between -100 and 100 would be useful in getting a better look at the graph.

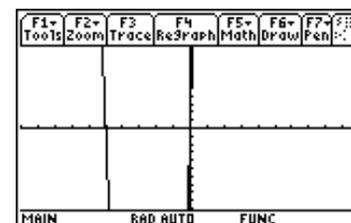


Figure 40

Set the following Window (using green \blacklozenge , F2).

xmin=-10 xmax=10 xscl=1 ymin=-100 ymax=100 yscl=10

Figure 41 shows the result.

If this were a parabola, we would stop here. But it is a cubic. So it must have another turning point. Is it to the left or right? One thing is definite: the turning point must have a value above the top of the graph, above $y=100$. So it would be a good idea to expand the graph to the left and right and increase ymax.

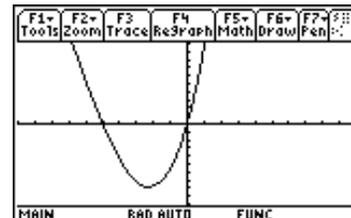


Figure 41

Set the following Window.

xmin=-20 xmax=20 xscl=5 ymin=-100 ymax=500 yscl=50

The result is Figure 42. This could be improved slightly (by having x go from -20 to 5 and y go from -100 to 300), but it fine as it is since it shows everything that is desired.

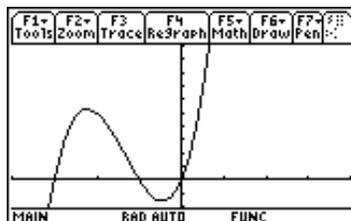


Figure 42

F5 (Math) can now be used to find the two turning points. Choice 4 (Maximum) reveals the turning point on the left is $(-11.08, 264.08)$. Choice 3 (Minimum) reveals the turning point on the right is $(-2.26, -78.89)$.

The zeros of the function can be found graphically or algebraically. You should know how to do this both ways. In the Graphing window select F5 (Math) and select choice 2 (Zero). Observe that once again you get the lower bound - upper bound questions. There are three places where the function crosses the x -axis (zeros). First move the cursor to the left of the left most zero and press ENTER for the lower bound. Then move the cursor to the right (but not beyond the next zero) and press ENTER for the upper bound. The calculator indicates the zero is $(-15, 0)$. Repeat this procedure for the other two zeros to obtain $(-5, 0)$ and $(0, 0)$.

Now obtain these zeros algebraically.

Press HOME to return to the home screen.

Press F2 (Algebra) choice 1 (solve).

Finish entering the following command.

$\text{solve}(x^3+20x^2+75x=0,x)$ ENTER

Figure 43 is the result.

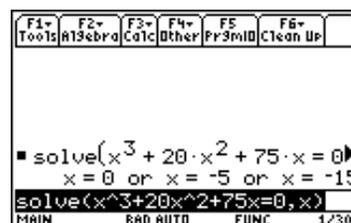


Figure 43

The final answer to the problem now is the following.

Figure 44 shows the graph.

The turning points are: $(-11.08, 264.08)$, a maximum
and $(-2.26, -78.89)$, a minimum.

The zeros are -15, -5 and 0.

(Note that the zeros can be obtained by factoring as follows:

$$0 = x^3 + 20x^2 + 75x = x(x^2 + 20x + 75) = x(x + 5)(x + 15)$$

so that $x = 0$, $x = -5$ (from $x + 5 = 0$) and $x = -15$.)

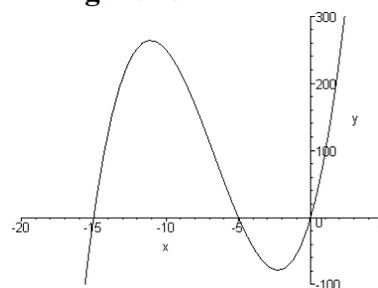


Figure 44

Example: Find the points of intersection of $y = x^3 + 20x^2 + 75x$ and $y = 30x + 200$

a) graphically and b) algebraically.

Graphical Solution: The previous example provides the graph of the first function, the cubic. So use the same graphing window $x_{\min}=-20$ $x_{\max}=20$ $x_{\text{scl}}=5$ $y_{\min}=-100$ $y_{\max}=500$ $y_{\text{scl}}=50$. It is assumed you have y_1 already entered in the TI-89.

Go to the $y=$ screen and enter $y_2 = 30x + 200$.

Now look at the graph. (Green \blacklozenge , F3)

The result is Figure 45.

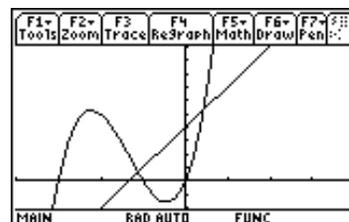


Figure 45

Two points of intersection are visible. But there should also be a third point of intersection below the current screen on the left. To see it, change y_{\min} from -100 to -500.

The result is Figure 46.

Now all three points of intersection are visible.

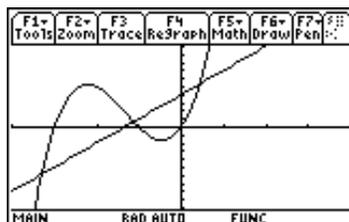


Figure 46

Press F5 (Math) choice 5 (intersection).

The result is Figure 47.

It does not ask for a lower bound. It asks for the 1st curve. The reason it does this is because there is the possibility of having more than two graphs displayed. Notice the "1" in the upper right corner and that the cursor appears on the cubic, which is the graph of y_1 .

Press ENTER to select y_1 as the first curve.

The upper right corner now has "2" appearing and the cursor is on the straight line, the graph of y_2 .

Press ENTER to select y_2 as the second curve.

Now the screen shows the usual question: "Lower bound?"

Move the cursor to the left of the lower left point of intersection.

Press ENTER.

Next, in response to "Upper bound?" move the cursor to the right of the point of intersection (but not beyond the next one).

Press ENTER.

Figure 48 appears. The point of intersection is $(-16.55104, -296.5313)$.

Doing the same for the other two points of intersection produces the additional two points of intersection $(-5.604898, 31.853065)$ and $(2.1559412, 264.67824)$.

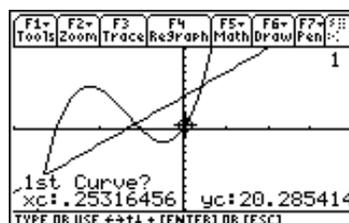


Figure 47

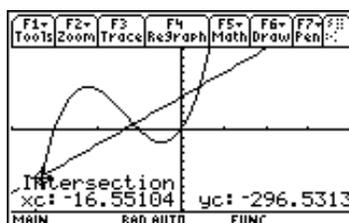


Figure 48

Algebraic Solution: Go to the home screen and enter the following command:

$\text{solve}(x^3+20x^2+75x=30x+200,x)$

to obtain the solutions $x = 2.1559412$ or $x = -5.6048978$ or $x = -16.551043$.

Now enter $30x+200|x=2.1559412$ to obtain the value of y , $y = 264.67824$.

Doing the same for the other two values of x produces the three points of intersection found by graphical means.

FUNCTIONS WITH VERTICAL ASYMPTOTES.

Example: Consider $y = f(x) = \frac{4}{5x - 15}$. It is undefined at $x = 3$ since $y = f(3) = \frac{4}{0}$.

The following table shows the values of y as you get closer to $x = 3$ on the left.

x	2	2.5	2.9	2.99	2.999	2.9999	2.99999	2.999999
y	-0.8	-1.6	-8	-80	-800	-8000	-80,000	-800,000

Clearly, each time another 9 is appended another 0 gets appended to the result. So, as x gets closer and closer to 3 on the left, y gets larger and larger in size as a negative number.

That is, $y \rightarrow -\infty$ (negative infinity).

The following table shows the values of y as you get closer to $x = 3$ on the right.

x	3.000001	3.00001	3.0001	3.001	3.01	3.1	3.5	4
y	800,000	80,000	8000	800	80	8	1.6	0.8

As x gets closer and closer to 3 on the right, y gets larger and larger in size as a positive number.

That is, $y \rightarrow \infty$ (positive infinity).

However, if the function is graphed using ZoomStd, the result is Figure 49.

The graph shown is incorrect. It makes it look as if the y value never goes below -10 nor above 5. It also makes it look as if the portion of the graph to the left of $x = 3$ is joined to the portion to the right of $x = 3$. If this were true, y would be defined at $x = 3$, and it is not. The reason the calculator produced the incorrect graph is due to the fact that it mindlessly plotted a lot of points and then connected them with line segments. Figure 50 shows the points that the calculator connected. After y was calculated at $x = 2.9113924$ to be -9.028571 , it next calculated the value of y at $x = 3.164557$ to be 4.8615385 (you can use Trace to see this). Since it was not aware of the problem at $x = 3$, it just connected those two points (but it should not have done this). You have to supply the thinking that corrects this.

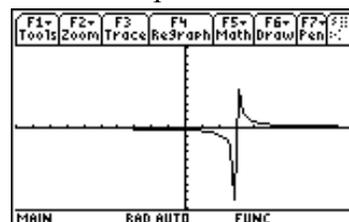


Figure 49

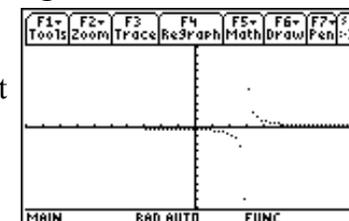


Figure 50

A vertical asymptote ($y \rightarrow -\infty$ or $y \rightarrow \infty$ on each side of $x = a$) exists at $x = a$ when the denominator of an expression is zero but the numerator is not, as is true in this case. You may then use the graph as a guide to the direction. In the current example you first identify the vertical asymptote by solving $5x - 15 = 0$. It is $x = 3$. The numerator 4 is not zero at $x = 3$. Examining Figure 45, you then conclude $y \rightarrow -\infty$ on the left and $y \rightarrow \infty$ on the right. Then draw the correct graph shown in Figure 51.

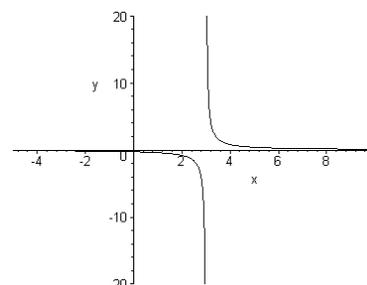


Figure 51

VERTICAL ASYMPTOTE: A VALUE OF x , $x = a$, WHERE $y \rightarrow -\infty$ or $y \rightarrow \infty$ ON THE RIGHT AND/OR LEFT SIDE OF $x = a$.

FACT: A RATIONAL FUNTION $y = f(x) = \frac{P(x)}{Q(x)}$ HAS VERTICAL ASYMPTOTES AT

THOSE VALUES OF x WHERE $Q(x) = 0$ BUT $P(x) \neq 0$ (i.e. the denominator is 0 but the numerator is not).

Example: Graph $y = f(x) = \frac{x}{x^2 - 9}$

Solution: This is a rational function.

The denominator is $x^2 - 9$.

Solving $x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = -3$ or 3 .

For $x = -3$ the numerator (x) is not 0; also for $x = 3$.

So $x = -3$ and $x = 3$ are vertical asymptotes.

Graphing the function with the TI-89 using ZoomStd results in Figure 52.

Since the two squiggles at $x = -3$ and $x = 3$ really should be vertical asymptotes, this graph must be corrected to look like Figure 53, the correct answer.

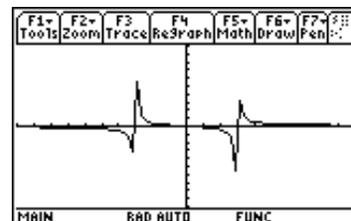


Figure 52

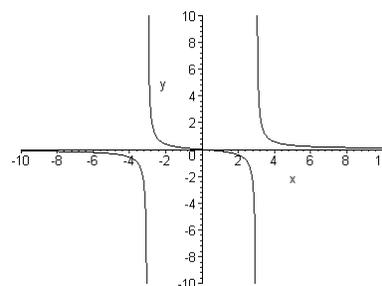


Figure 53

Example: It is known that for a logarithmic function such as

$$y = f(x) = \frac{x \ln(2x + 8)}{3}$$

a vertical asymptote exists for the value of x at which $2x + 8 = 0$.

Thus $x = -4$ is a vertical asymptote. Figure 54 shows the TI-89 graph of this function. The function is not defined for $x \leq -4$.

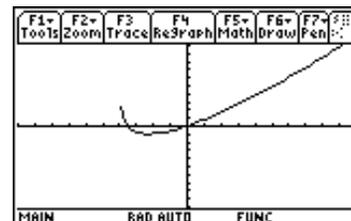


Figure 54

Knowing that $x = -4$ should appear as a vertical asymptote, the graph should be corrected to look like Figure 55.

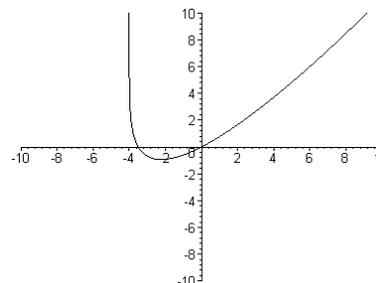


Figure 55

MORE ALGEBRA AND COMPLEX NUMBERS

MORE FACTORIZATION.

Press F2 (Algebra) and select choice 2:factor.

Press $3x^5 - x^4 - 3x^3 + x^2 - 6x + 2$ ENTER.

Figure 56 shows the result.

$$3x^5 - x^4 - 3x^3 + x^2 - 6x + 2 = (3x - 1)(x^2 - 2)(x^2 + 1)$$

The factor command shown only factors over the integers.

So although $x^2 - 2 = (x + \sqrt{2})(x - \sqrt{2})$, this is not done.

The TI-89 can perform this factorization also.

Press the right arrow cursor to remove the highlighting from the command line, positioning the cursor at the end of the line, and then press the left cursor arrow to place the cursor just before the last parenthesis.

Insert \sqrt{x} and press ENTER.

Figure 57 shows the result.

$$3x^5 - x^4 - 3x^3 + x^2 - 6x + 2 = (3x - 1)(x + \sqrt{2})(x - \sqrt{2})(x^2 + 1)$$

It is also known that $i^2 = -1$ so that $(x + i)(x - i) = x^2 - i^2 = x^2 - (-1) = x^2 + 1$.

Thus $x^2 + 1$ factors over the complex integers as $(x + i)(x - i)$.

The TI-89 can perform this factorization also.

Press the left arrow cursor to remove the highlighting from the command line, positioning the cursor at the beginning of the line, and then press the purple alpha key followed by “)” in order to insert “c” at the beginning of the command line.

Move the cursor towards the end of the line and delete \sqrt{x} .

Press ENTER. Figure 58 shows the result.

Thus $\text{cfactor}(3x^5 - x^4 - 3x^3 + x^2 - 6x + 2)$ ENTER yields

$$3x^5 - x^4 - 3x^3 + x^2 - 6x + 2 = (3x - 1)(x^2 - 2)(x + i)(x - i)$$

The TI-89 can be used to put the commands shown together in order to factor an expression completely over all complex numbers.

Insert \sqrt{x} just before the last parenthesis and press ENTER.

Figure 59 shows the result.

$$3x^5 - x^4 - 3x^3 + x^2 - 6x + 2 = (3x - 1)(x + \sqrt{2})(x - \sqrt{2})(x + i)(x - i)$$

Note: The cfactor command can also be accessed by pressing F2 (Algebra), pressing the down cursor arrow until A:Complex appears, pressing the right cursor arrow, and then select choice 2:cfactor.

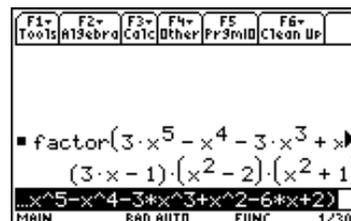


Figure 56

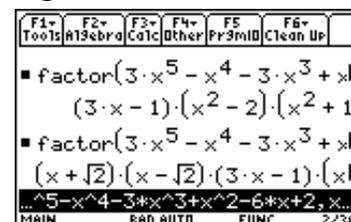


Figure 57

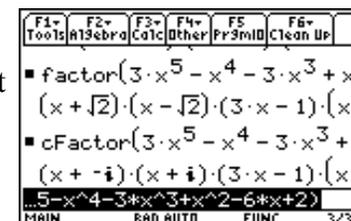


Figure 58



Figure 59

Summary: **factor(expression)** factors expression over the integers. (Usual factorization)
factor(expression,x) factors *expression* over the real numbers.
cfactor(expression) factors *expression* over the complex integers.
cfactor(expression,x) factors *expression* over the complex numbers.
 (Use this factorization to factor a polynomial into linear factors.)

Example: Factor $x^6 + 2x^5 - x^4 - 4x^3 - 17x^2 - 30x - 15$ completely into linear factors.

Solution: Press F2 (Algebra) choice 2 (factor)

Press the left cursor arrow repeatedly to return to the beginning of the command line.

Press the purple alpha key and then the key with the purple C above it (left parenthesis).

Press the right cursor arrow repeatedly to return to the end of the command line.

Press $x^6+2x^5-x^4-4x^3-17x^2-30x-15,x$ ENTER

You have entered `cfactor(x^6+2x^5-x^4-4x^3-17x^2-30x-15)`

So $x^6 + 2x^5 - x^4 - 4x^3 - 17x^2 - 30x - 15 = (x + 1)^2(x + \sqrt{5})(x - \sqrt{5})(x + \sqrt{3}i)(x - \sqrt{3}i)$

Note: Similarly, placing a c in front of the algebra command `solve(equation,variable)` finds all solutions, both real and complex, to an *equation*.

LONG DIVISION: propFrac(expression)

Example: Use long division to divide $12x^3 - 11x^2 + 13x + 19$ by $4x + 3$.

Solution: Use choice 7:propFrac (proper fraction) on the algebra menu to enter the following command.

`propFrac((12x^3-11x^2+13x+19)/(4x+3))`

Figure 60 shows the result.

$$\frac{12x^3 - 11x^2 + 13x + 19}{4x + 3} = 3x^2 - 5x + 7 - \frac{2}{4x + 3}$$

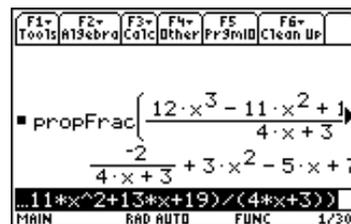


Figure 60

Press right cursor



Figure 66

Press 2 (Matrix)



Figure 67

Press down cursor twice so that the cursor blinks in the variable rectangle.

Press A (key with = on it)

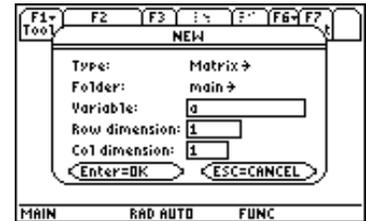


Figure 68

Press down cursor, 2, down cursor, 3

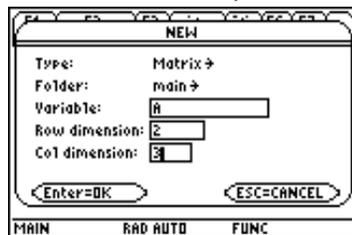


Figure 69

Press ENTER twice

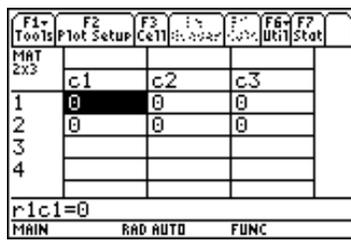


Figure 70

Press 3, ENTER

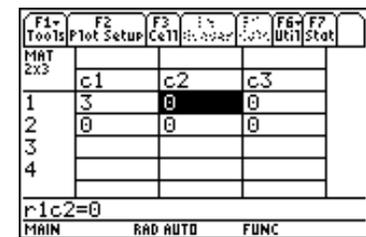


Figure 71

Example: Given the matrices $A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 2 & -3 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 9 & 5 \\ 1 & 4 & 3 \\ 3 & 14 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}$ find:

- a) A^{-1} b) B^{-1} c) $2A - 3B$ d) $A^{-1}C$

Solution: Enter the three matrices A, B and C. It is very likely that you encountered the screen shown in Figure 77 when you tried to enter A. That is because you must first delete the previous matrix entered that was also called A as was shown at the beginning of the section on matrices: F6 (i.e. 2nd F1), ENTER, ENTER.

To find A^{-1} , simply enter A^{-1} on the command line of the home screen, i.e. alpha, ↑, =, ^, (, (-),), 1.
Figure 78 is the result.

$$\text{Hence, } A^{-1} = \begin{bmatrix} 3/7 & 1/7 & 2/7 \\ 0 & 1 & 1 \\ -1/7 & 2/7 & 4/7 \end{bmatrix}$$

To find B^{-1} enter B^{-1} in the same way.

Figure 79 is the result.

This means the matrix B does not have an inverse.

To find $2A - 3B$ simply enter that expression on the command line, i.e. 2, alpha, ↑, =, -, 3, alpha, ↑, (, ENTER

$$\text{Hence, } 2A - 3B = \begin{bmatrix} -2 & -27 & -17 \\ -5 & -8 & -15 \\ -7 & -44 & -15 \end{bmatrix} \quad (\text{See Figure 80})$$

To find $A^{-1}C$, enter that expression on the command line, i.e. alpha, ↑, =, (, (-),) 1, *, alpha, ↑,), ENTER

$$\text{Hence, } A^{-1}C = \begin{bmatrix} 1 \\ 8 \\ 4 \end{bmatrix} \quad (\text{See Figure 81})$$

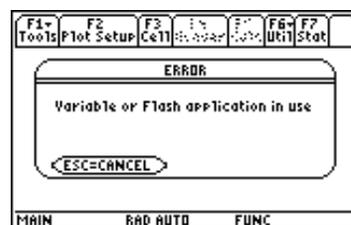


Figure 77

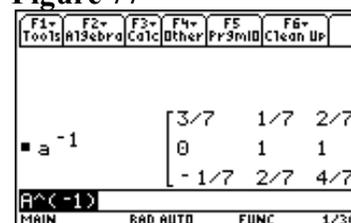


Figure 78

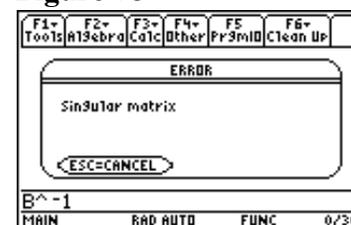


Figure 79

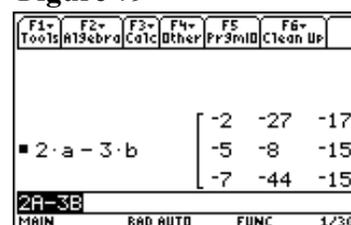


Figure 80

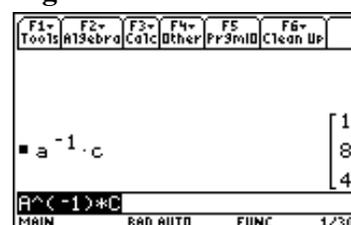


Figure 81

CALCULUS

LIMITS: limit(expression, variable, value) and limit(expression, variable, value, ±1).

This function is found on the Home screen by using the F3 Calc key. The calculus menu that appears when F3 is pressed is shown in Figure 82. The limit function is selected by either pressing 3 or using the down arrow cursor to highlight choice 3:limit and then pressing ENTER.



Figure 82

$$\lim_{x \rightarrow c} f(x) = \text{limit}(f(x), x, c)$$

Example: Find $\lim_{x \rightarrow 16} \frac{x-4}{16-\sqrt{x}}$

Solution: Select the limit function as indicated above. Complete the command line as follows and press ENTER:

$$\text{limit}((x-4)/(16-\sqrt{x}), x, 16)$$

The answer is 1 as shown in Figure 83.

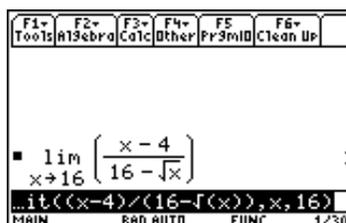


Figure 83

Example: Find $\lim_{\Delta x \rightarrow 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$

Solution: In order to use the calculator for this problem it is best to think of Δx as a single letter such as t . That is,

$$\lim_{t \rightarrow 0} \frac{\frac{1}{x+t} - \frac{1}{x}}{t}$$

Now enter on the command line the following:

$$\text{limit}((1/(x+t)-1/x)/t, t, 0)$$

The result is $-1/x^2$ as shown in Figure 84.

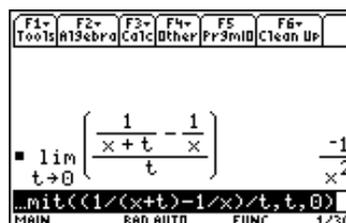


Figure 84

Example: Find $\lim_{x \rightarrow \infty} \frac{5-8x^2}{4x^2-3x+1}$

Solution: Notice “∞” appears in green above the catalog key.

So infinity, ∞, is entered by pressing green ♦, catalog.

So $\text{limit}((5-8x^2)/(4x^2-3x+1), x, \infty) = -2$ as shown in Figure 85.

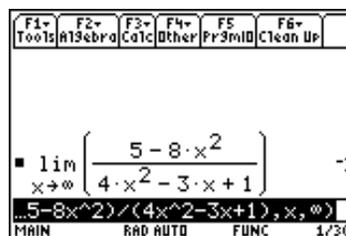


Figure 85

One sided limits: An extra “,1” is added for right sided limits and an extra “,-1” for left sided ones.

$$\lim_{x \rightarrow c^+} f(x) = \text{limit}(f(x), x, c, 1)$$

$$\lim_{x \rightarrow c^-} f(x) = \text{limit}(f(x), x, c, -1)$$

Example: Find a) $\lim_{x \rightarrow 0^+} \frac{|x|}{x}$ and b) $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$

Solution: The absolute value function can be obtained by pressing the catalog key and then “=” to obtain the functions beginning with the letter “a”. The first such function is “abs(“, absolute value. See Figure 86. Pressing ENTER then moves the function to the command line.

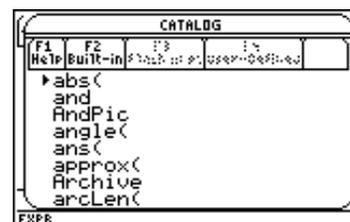


Figure 86

a) Clear the entry line on the home screen.
 Press F3 (Calc) choice 3 (Limit)
 Press catalog, =, ENTER (for “abs(“)
 Finish the command by entering x)/x,x,0,1)
 Figure 87 is the result.
 So the first answer is 1.

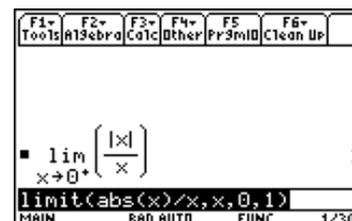


Figure 87

b) Editing the command line by inserting a negative sign in front of the one produces $\text{limit}(\text{abs}(x)/x, x, 0, -1) = -1$

Example: Find a) $\lim_{x \rightarrow 5^+} \frac{1-x}{2x-10}$ and a) $\lim_{x \rightarrow 5^-} \frac{1-x}{2x-10}$

Solution: a) $\text{limit}((1-x)/(2x-10), x, 5, 1) = -\infty$
 b) $\text{limit}((1-x)/(2x-10), x, 5, -1) = \infty$

DERIVATIVES: $d(\text{function, variable})$ and $d(\text{function, variable}, 2)$.

The differentiation operator d appears in yellow above the “8” key. It is accessed by pressing the yellow 2nd key and then 8. It can also be obtained in the Home screen by pressing F3 (Calc) followed by selecting choice 1 (differentiate). The purple D appearing above the comma key cannot be used for this purpose.

If $y = f(x)$, then $d(f,x) = f'(x)$, the derivative of $f(x)$
and $d(f,x,2) = f''(x)$, the second derivative of $f(x)$

Example: Given the function $y = f(x) = \frac{x \ln(x)}{3x + 5}$

a) find $f'(x)$ and solve $f'(x) = 0$. b) find $f''(x)$ and solve $f''(x) = 0$.

Solution:

a) Enter $d((x*\ln(x))/(3x+5),x)$ on the command line.

Figure 88 shows the result. $f'(x) = \frac{5\ln(x)}{(3x+5)^2} + \frac{1}{3x+5}$

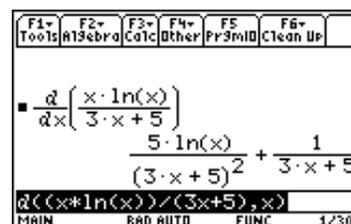


Figure 88

Press F2 (Algebra) choice 1 (solve).

Press the up arrow cursor once.

Figure 89 is the result.

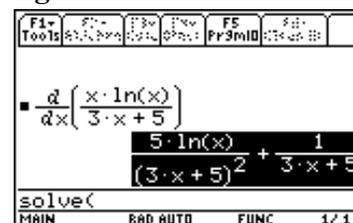


Figure 89

Press ENTER to paste the highlighted expression on the entry line.

Press =0,x) (to obtain solve(expression=0,x))

Press ENTER to get Figure 90.

At the bottom of the screen the words “Warning: More solutions may exist” appear. Ignore this whenever you see it.

The only solution to $f'(x) = 0$ is $x = 0.30614744$.

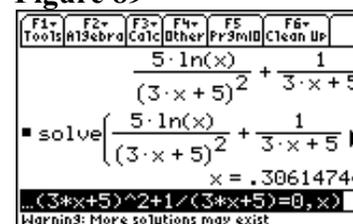


Figure 90

b) Everything done in part (a) is repeated in part (b) except the second derivative is used instead of the first derivative.

Press $d((x*\ln(x))/(3x+5),x,2)$ ENTER to obtain $f''(x) = \frac{-30\ln(x)}{(3x+5)^3} - \frac{3x-5}{x(3x+5)^2}$

Press F2 (algebra), choice 1 (solve), up cursor, ENTER, =0,x) ENTER.

The only solution to $f''(x) = 0$ is $x = 1.2927652$.

USING A GRAPH TO FIND TANGENT LINES, EXTREMA AND INFLECTION POINTS.

Once a function is graphed, the F5 (Math) menu in the graphing screen can be used to find tangent lines, extrema and inflection points. However, THE POINTS THAT MUST BE INVESTIGATED MUST BE FOUND BEFORE GRAPHING AND BE VISIBLE IN AN APPROPRIATE GRAPHING WINDOW.

Example: For the function in the previous example, $y = f(x) = \frac{x \ln(x)}{3x + 5}$, find the extrema, points of inflection, and graph it.

Solution:

In the $y=$ window enter the above function for $y1$.

Press F2 (Zoom) choice 6 (ZoomStd). The result is Figure 91.

(Note that $\ln(x)$ is only defined for $x > 0$. The denominator is 0 when $x = -5/3$, but this is not relevant to the problem since the function is only defined for $x > 0$ due to the $\ln(x)$.)

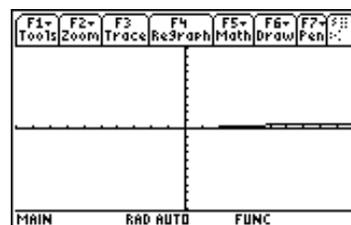


Figure 91

Before selecting the graphing window desired, you need to know the points that should show up in the window. For extrema, the points are the ones where $f'(x) = 0$ or is undefined. In the previous example it was determined that the only solution to $f'(x) = 0$ is $x = 0.30614744$. $f'(x)$ is undefined for $x \leq 0$, but so is $f(x)$. For points of inflection, the points of interest are the points where $f''(x) = 0$. In the previous example the only solution to $f''(x) = 0$ is $x = 1.2927652$. So only values of x between 0 and 2 are really of interest.

Press F3 (Trace) and move the cursor between $x = 0$ and $x = 2$.

The y values for these values of x are between -0.1 and 0.2 .

So change the graphing window to:

$x_{\min} = -1$ $x_{\max} = 3$ $x_{\text{scl}} = 1$ $y_{\min} = -0.1$ $y_{\max} = 0.2$ $y_{\text{scl}} = 0.1$

The result is Figure 92.

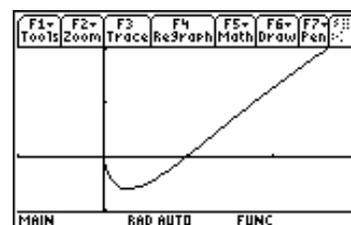


Figure 92

It is clear that $x = 0.30614744$ corresponds to a minimum. To verify this and find the corresponding value of y , Press F5 (Math) choice 3 (minimum), move the cursor to the left of the minimum and press ENTER for the lower bound, and then move the cursor to the right of the minimum and press ENTER for the upper bound. The result is Figure 93.

The minimum is $(0.3061473, -0.0612295)$.

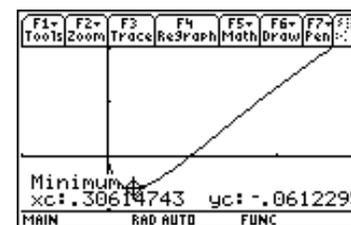


Figure 93

A point of inflection is not obvious on the graph. However, $x = 1.2927652$ is the only possible candidate. So press F5 (Math) choice 8 (inflection), move the cursor so that it is to the left of $x=1.2$ and press ENTER for the lower bound, and then move the cursor to the right of $x=1.4$ and press ENTER for the upper bound. The result is Figure 94.

The point of inflection is $(1.2927652, 0.03739015)$.

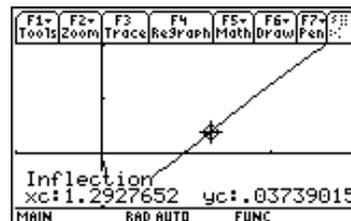


Figure 94

In order to actually see the change in concavity for the graph, you would need to see more of the graph on the right side (but this is not necessary to answer the question). So change the window to $xmin=-1$ $xmax=8$ $xsc1=1$ $ymin=-0.1$ $ymax=0.7$ $yscl=0.1$. The result is Figure 95.

It is now evident that the graph is concave up on the left and concave down on the right.

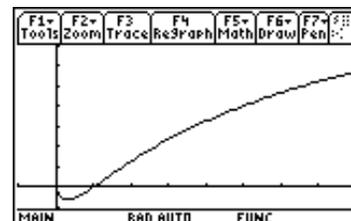


Figure 95

Example: Find the equation of the line tangent to $y = f(x) = \frac{15x}{x^2 + 3}$ at $(1, 3.75)$.

Solution: Graph the function in the ZoomStd window.

The result is Figure 96.

Since the $(1, 3.75)$ is in this window, we do not have to change it.

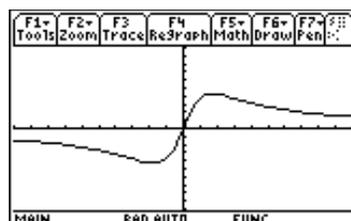


Figure 96

Press F5 (Math). You do not see a choice for "Tangent."

However, the down arrow next to the number 8 in the menu tells you that there is more to the menu. So press the down cursor until the menu appears as in Figure 97. Now move the cursor back so that choice A (Tangent) is highlighted. Press ENTER.

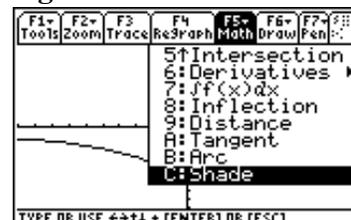


Figure 97

In response to the question "Tangent at?"

press 1 ENTER (to indicate the value of x is $x = 1$).

The result is Figure 98.

The equation of the tangent line appears at the bottom left.

The tangent line at $(1, 3.75)$ is $y = 1.875x + 1.875$.

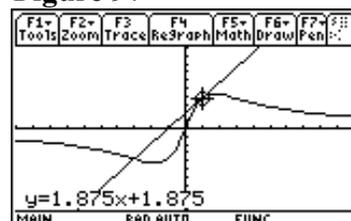


Figure 98

MISCELLANEOUS INFORMATION

General information concerning TI calculators can be found at <http://education.ti.com>.

CORRECTING THE STRANGE TOOLBAR MENU PROBLEM

At some point you might accidentally do something that makes the Home screen look like it does in Figure 99.

Press F7 (2nd F2).

Select 3: CustmOff.

Press ENTER.

The toolbar now appears as before.

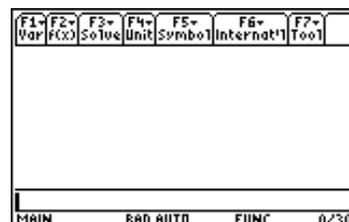


Figure 99

CORRECTING OVERWRITE MODE

By default, the calculator uses insertion mode. However, you might accidentally place the calculator in overwrite mode. If your calculator shows the blinking cursor as a rectangle instead of a vertical line and pressing a key overwrites the character, then this can be corrected by pressing 2nd ← (INS = insert toggles between insertion and overwrite mode).