MATH 2003
TEST A

This part of the exam is to be done without a calculator.

1. Find \( \lim_{x \to 2} \frac{x^2 - x - 2}{x - 2} \)
   a) 3   b) 4   c) 0   d) 1   e) \( \infty \)

2. Given \( f(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ x - 1 & \text{if } x \geq 0 \end{cases} \), find \( \lim_{x \to 0} f(x) \) if it exists.
   a) 0   b) 1   c) -1   d) it does not exist   e) 2x

3. Find the horizontal asymptote for the graph of \( y = \frac{2+x}{1-x} \)
   a) \( y = 0 \)   b) \( y = 1 \)   c) \( y = 3 \)   d) \( y = 2 \)   e) \( y = -1 \)

4. The equation of the straight line through the point \((-6, 4)\) with \(x\)-intercept 3 is
   a) \( y = -\frac{4}{3}x + 4 \)   b) \( y = \frac{4}{3}x - 4 \)   c) \( y = -\frac{1}{6}x + 3 \)   d) \( y = \frac{1}{6}x + 3 \)
   e) \( y = -\frac{4}{9}x + \frac{4}{3} \)

5. What is the \(x\)-intercept (zero) of the graph of \( y = \frac{\frac{3}{8}x^2 - 8}{1} \)?
   a) -16   b) -8   c) \( \frac{1}{16} \)   d) 16   e) 512

6. ABC Inc. buys a new truck for $32,000. The truck will have a scrap value of $8,000 after 8 years. If they use the straight-line method for depreciating the truck, find the value after 2 years.
   a) $20,000   b) $22,000   c) $24,000   d) $26,000   e) $28,000

7. Find the center of a circle given by the equation \( x^2 + y^2 + 2x - 2y = 0 \)
   a) \((0, 0)\)   b) \((-1, 1)\)   c) \((1, -1)\)   d) \((2, -2)\)   e) \((-2, 2)\)
8.

The graph of the function $f$ and a table of values for the function $g$ are shown above. What is the value of $f(g(0))$?

a) -4  b) -2  c) 0  d) 2  e) 4

9. The axis of symmetry of $y = x^2 - 4x + 5$ is:

a) -4  b) $y = 5$  c) $y = 2$  d) $x = -4$  e) $x = 2$

10. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =

a) $\frac{3x-3}{\sqrt{2x-3}}$

b) $\frac{x}{\sqrt{2x-3}}$

c) $\frac{1}{\sqrt{2x-3}}$

d) $\frac{-x+3}{\sqrt{2x-3}}$

e) $\frac{5x-6}{2\sqrt{2x-3}}$
11. If \( f(x) = -x^3 + x + \frac{1}{x} \), then \( f'(-1) = \)
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17. When solving a system of linear equations using the Gauss-Jordan Elimination Method the matrix below was obtained. What can be concluded about the solution of the system?

\[
\begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 3 & 4 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

a) No solution  
b) Infinite number of solutions with \( x = 2, \ y = 4, \ z = z \)  
c) Infinite number of solutions with \( x = 2, \ y = 4 - 3z, \ z = z \)  
d) Unique solution \( x = 2, \ y = 3, \ z = 4 \)  
e) Unique solution \( x = 2, \ y = 4, \ z = 0 \)

18. Find \( f''(x) \) for \( f(x) = \frac{-5}{(x+4)^3} \)

a) \( \frac{15}{(x+4)^2} \)  
b) \( \frac{15}{(x+4)^4} \)  
c) \( \frac{-3}{(x+4)^4} \)  
d) \( \frac{5}{3(x+4)^4} \)  
e) 0

19. If the points of a certain set do not lie close to a line, then which of the following is the best possible value of the correlation coefficient, determined for the line of best fit?

a) 0.98  
b) 0.5  
c) 0.02  
d) -0.97  
e) 2

20. The figure to the left shows the graph of the polynomial function \( g \). Which of the following could define \( g(x) \)?

a) \( g(x) = x^3 - 4 \)  
b) \( g(x) = x^3 - 4x \)  
c) \( g(x) = -x^3 + 4x \)  
d) \( g(x) = x^4 - 4x^2 \)  
e) \( g(x) = -x^4 + 4x^2 \)
21. The graph in the xy-plane of which of the following equations is a parabola?
   a) $2xy = 1$
   b) $x^2 - 2x + 3y = 1$
   c) $x^2 - 4x + y^2 - y = 1$
   d) $x^2 - y^2 + 6y = 1$
   e) $(x - 2)^2 = y^2$

22. Evaluate $\frac{f(x+h) - f(x)}{h}$ for $f(x) = x^2 + 3$
   a) 1  
   b) $2x + h + 3$
   c) $2x + h$
   d) $2x$
   e) $x + h$

23. Find all the points at which the graph of $f(x) = 3x^4 - 4x^3 + 5$ has horizontal tangent lines.
   a) $(0, 5)$ and $(1, 4)$  
   b) $(0, 5)$ and $\left( \frac{2}{3}, \frac{119}{27} \right)$  
   c) $(0, 5)$ only  
   d) $(0, 0)$ and $(1, 0)$  
   e) $(1, 4)$ only

24. If a profit equation is $P(x) = -5x^2 + 10x + 20$, then the maximum profit is
   a) 10  
   b) 20  
   c) 15  
   d) 25  
   e) Impossible to tell

25. Let $f$ be the function defined by $f(x) = -x^2$. The graph of the function $g$ in the xy-plane is obtained by first translating the graph of $f$ horizontally 3 units to the left and then vertically translating this result 2 units up. What is the value of $g(-2)$?
   a) -7  
   b) -3  
   c) 0  
   d) 1  
   e) 3
MATH 2003
TEST A

Some of the questions on this part of the exam require a calculator

26. \( f(x) = x(x-1) \)
    \( g(x) = x \)

The functions \( f \) and \( g \) are defined above. What are all values of \( x \) for which
\( f(x) < g(x) \)?

a) \( x < 0 \) or \( x > 1 \)

b) \( x < 0 \) or \( x > 2 \)

c) \( 0 < x < 1 \)

d) \( 0 < x < 2 \)

e) \( 1 < x < 2 \)

27. Let \( f \) be the function defined by
\[ f(x) = \begin{cases} 
5-x, & x \leq 3 \\
-x^2+10x-8, & x > 3 
\end{cases} \]

Find all values of \( x \) for which \( f(x) = 4 \)

a) \( x = 1 \) only  

b) \( f(x) \) never equals 4  

c) \( x = 1, x = 1.394 \) and \( x = 8.606 \)

d) \( x = 1 \) and \( x = 8.606 \)

e) \( x = 1.394 \) and \( x = 8.606 \)

28. If \( \begin{bmatrix} -2 & 8 & 1 \\ 0 & 1 & 0 \\ 3 & -12 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} \) is represented by the matrix equation
\( AX = B \) find the solution \( X \).

a) \( \begin{bmatrix} 41 \\ 5 \\ -60 \end{bmatrix} \)

b) \( \begin{bmatrix} 10 \\ 9 \\ 7 \end{bmatrix} \)

c) \( \begin{bmatrix} 7 \\ -23 \\ 4 \end{bmatrix} \)

d) \( \begin{bmatrix} 24 \\ 5 \\ 9 \end{bmatrix} \)

e) No solution exists
29. Solve the system of equations

\[ \begin{align*}
  x + z &= 3 \\
  x + y - 2z &= 10 \\
  y - 3z &= 8
\end{align*} \]

a) \((3, 7, 1)\)  \quad b) \((3 - z, 7 + 3z, 0)\)  \quad c) \((3 - z, 7 + 3z, z)\)  \quad d) \((3 - z, 7 + 3z, 1)\)  \quad e) No solution exists

30. The radius of a circle is increasing at the rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the radius is 10 meters?

a) \(0.04\pi \text{ m}^2/\text{sec}\)  \quad b) \(0.4\pi \text{ m}^2/\text{sec}\)  \quad c) \(4\pi \text{ m}^2/\text{sec}\)  
\quad d) \(20\pi \text{ m}^2/\text{sec}\)  \quad e) \(100\pi \text{ m}^2/\text{sec}\)

31. Which of the following is an equation if the line tangent to the graph of \(f(x) = x^4 + 2x^2\) at the point where \(f'(x) = 1\)?

a) \(y = 8x - 5\)  \quad b) \(y = x + 7\)  \quad c) \(y = x + 0.763\)  
\quad d) \(y = x - 0.122\)  \quad e) \(y = x - 2.146\)

32. In your computer store you make $1012 per computer sold. Your costs are given by the equation \(C(x) = 4x^2 + 4x - 80\), where \(x\) is the number of computers sold. How many computers should you sell to maximize the profit?

a) 2  \quad b) 126  \quad c) 128  \quad d) 127  \quad e) The more sold the greater the profit.

33. Find the limit: \(\lim_{x \to 3} \frac{x^2 - 3x + 2}{x^2 - 5x + 6}\)

a) \(\frac{1}{3}\)  \quad b) \(\infty\)  \quad c) \(-\infty\)  \quad d) 1  \quad e) None of these
34. A sample of 5 adult men gave the following data on their heights and weights. Find the linear regression equation and round the coefficients of the regression equation to 3 decimal places.

<table>
<thead>
<tr>
<th>Height (inches)</th>
<th>Weight (pounds)</th>
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</thead>
<tbody>
<tr>
<td>62</td>
<td>130</td>
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<tr>
<td>65</td>
<td>145</td>
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<td>67</td>
<td>140</td>
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<td>68</td>
<td>155</td>
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<tr>
<td>72</td>
<td>168</td>
</tr>
</tbody>
</table>

a) \( y = 3.697x - 99.365 \)
b) \( y = -99.365x + 3.697 \)
c) \( y = 0.240x + 31.420 \)
d) \( y = 31.420x + 0.240 \)
e) \( y = -3.697x - 99.365 \)

35. The total profit that a company has obtained since it started a business is given by \( P(t) = 1500t - 3\sqrt{t} - 50,000 \) where \( t \) is the number of months since the company started business. Find the average rate of change in profit with respect to time between 3 months after the company started and 7 months after the company started.

a) $5,997.26
b) -$42,506.60
c) $2,998.57
d) $1,499.28
e) $1,499.31
<table>
<thead>
<tr>
<th>Answers to Test A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) a      2) d  3) e  4) e  5) d</td>
</tr>
<tr>
<td>6) d      7) b  8) b  9) e  10) a</td>
</tr>
<tr>
<td>11) d     12) d  13) b  14) a  15) a</td>
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<tr>
<td>16) b     17) c  18) b  19) c  20) c</td>
</tr>
<tr>
<td>21) b     22) c  23) a  24) d  25) d</td>
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<tr>
<td>26) d     27) d  28) d  29) e  30) c</td>
</tr>
<tr>
<td>31) d     32) b  33) c  34) a  35) e</td>
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