

Entrepreneurial Finance and the Non-diversifiable Risk*

Hui Chen[†] Jianjun Miao[‡] Neng Wang[§]

September 12, 2008

Preliminary Version

Abstract

Entrepreneurs face significant non-diversifiable idiosyncratic business risks. In a dynamic incomplete-markets model of entrepreneurial finance, we show that such risks have important implications for their interdependent consumption/saving, portfolio choice, financing, investment, and endogenous default/cash-out decisions. Even though more risk-averse entrepreneurs default earlier for given debt service, they choose higher leverage *ex ante* for diversification benefits. Entrepreneurs demand not only a systematic risk premium but also an idiosyncratic risk premium due to the lack of diversification. We derive an analytical formula for the idiosyncratic risk premium whose key determinants are risk aversion, idiosyncratic volatility and the sensitivity of entrepreneurial value of equity with respect to cash flow. An entrepreneur's option to use external equity in the future increases his diversification benefits and private value of firm, but crowds out/lowers the value of diversification via external risky debt. When an entrepreneur chooses one of mutually exclusive projects with different idiosyncratic volatilities after debt is in place, the effect of risk aversion may dominate the risk-shifting incentives. Only entrepreneurs with a low level of risk aversion engage in risk shifting activities.

Keywords: Default, diversification benefits, entrepreneurial risk aversion, incomplete markets, private equity premium, hedging, capital structure, cash-out option, precautionary saving

JEL Classification: G11, G31, E2

*We thank John Cox and Debbie Lucas for comments.

[†]MIT Sloan School of Management, 50 Memorial Drive, Cambridge, MA 02142. Email: huichen@mit.edu. Tel.: 617-324-3896.

[‡]Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215. Email: miaoj@bu.edu. Tel.: 617-353-6675.

[§]Columbia Business School and National Bureau of Economic Research. Address: Columbia Business School, 3022 Broadway, Uris Hall 812, New York, NY 10027. Email: neng.wang@columbia.edu. Tel.: 212-854-3869.

1 Introduction

Entrepreneurship plays an important role in fostering innovation and economic growth (Schumpeter (1934)). Entrepreneurial investment activities are quite diverse, ranging from the creation of the state-of-the-art high-tech products to daily operations of small business (e.g. restaurants). Entrepreneurial businesses may differ from each other, but share one important common feature: entrepreneurs are often exposed to non-diversifiable idiosyncratic risks from their investment projects. For reasons such as incentive alignment and informational asymmetry between entrepreneurs and financiers, entrepreneurs typically hold an undiversified portfolio, and thus bear non-diversifiable idiosyncratic risks. Moskowitz and Vissing-Jorgensen (2002) document that about 75 percent of all private equity is owned by households for whom it constitutes at least half of their total net worth. Moreover, households with entrepreneurial equity invest on average more than 70 percent of their private holdings in a single private company in which they have an active management interest. Hall and Woodward (2008) document that the idiosyncratic risks faced by entrepreneurs of startups are so large that, even with moderate risk aversion, they will be better off passing up projects with high expected payoffs.

The significant lack of diversification invalidates the standard finance textbook valuation analysis designed for firms owned by diversified investors. Unlike the standard argument that a firm's idiosyncratic risks carry no risk premium for investors holding a diversified portfolio, an entrepreneur's non-diversifiable position in his own investment project makes his business decisions (financing and project choice) and his household decisions (consumption, saving, and asset allocation) interdependent. This interdependence arises in our model because markets are no longer complete an entrepreneur. The standard two-step complete-markets analysis (i.e. first value maximization and then optimal consumption allocation) no longer applies. This non-separability between value maximization and consumption smoothing has potentially important implications on real economic activities and the pricing of financial claims that an entrepreneur issues to finance his investment activities.

To the best of our knowledge, this paper provides the first dynamic incomplete-markets trade-off model of capital structure for entrepreneurial firms. We consider an infinitely-lived risk-averse entrepreneur who derives utility from his intertemporal consumption. He has both liquid financial wealth and an illiquid (entrepreneurial) investment project, which he sets up as a firm with limited

liability (the entrepreneurial firm). He can invest his liquid financial wealth in both a risk-free asset earning a constant rate of return, and the market portfolio yielding a constant risk premium as in the classic consumption/portfolio choice problem (Merton (1971)). The entrepreneurial firm's investment project generates a stream of stochastic cash flows, which are imperfectly correlated with the market portfolio. The entrepreneur trades dynamically in the market portfolio to hedge the systematic component of his business risk. However, the entrepreneur cannot fully hedge his project risk because his business risk is imperfectly correlated with the market portfolio (i.e. incomplete markets), and hence faces non-diversifiable idiosyncratic risks.

To reduce his idiosyncratic risk exposure, the entrepreneur uses both outside risky debt and equity to diversify his idiosyncratic risk. External equity financing can either take the form of initial public offering (IPO) or a direct sale of the firm to diversified investors. Our model incorporates the feature that issuing outside equity (either via direct placement or IPO) requires the entrepreneur to pay a fixed cost.¹ The fixed cost of issuing external equity generates the option value of waiting before accessing external equity. This prediction is consistent the empirical evidence that debt is the primary source of outside financing for small businesses (Heaton and Lucas (2004)).² Naturally, the entrepreneur only accesses external equity when the diversification benefits from cashing out is sufficiently high. The buyer of the firm often optimally readjusts the firm's leverage. We also incorporate the effect of leverage adjustment when the firm cashes out on the *ex ante* entrepreneur's capital structure decision and valuation for his firm.

After borrowing on the firm's balance sheet using the illiquid project as collateral, the entrepreneur has an endogenous default option. As in standard tradeoff models, the default option generates a conflict of interest between the entrepreneur and the lender. Unlike in standard corporate finance models, this default option allows the entrepreneur to walk away from his business with limited liability, hence providing an *ex ante* insurance to the entrepreneur against the firm's potential poor performance in the future. Default triggers costly liquidation as in standard tradeoff models of capital structure. In addition to standard implications of debt financing such as tax, financial distress and agency costs, the risky debt also offers diversification benefits for the entrepreneur. The entrepreneur chooses the amount of outside risky debt by trading off tax

¹Moreover, debt is potentially less information sensitive, and may be preferred in some settings with asymmetric information (Myers and Majluf (1984)). Our model does not explicitly model information asymmetry.

²The non-diversifiable idiosyncratic risk that entrepreneurs bear also has implications on the *ex ante* career choice. See Kihlstrom and Laffont (1979) and Rampini (2004) building on the ideas in Knight (1921).

implications/diversification benefits with financial distress and agency costs. We show that the entrepreneur's utility maximization problem (with interdependent consumption/saving, portfolio and default decisions) can be simplified to a problem of maximizing the entrepreneur's private value of the firm, which is given by the sum of the entrepreneur's private value of equity and the market value of debt he borrows. The entrepreneur's risk attitude and his exposure to the project's idiosyncratic risks have significant effects on the private valuation of the entrepreneurial firm and its equity position, and hence influence the firm's initial capital structure and subsequent default decisions.

Unlike the standard tradeoff models of capital structure, the natural measure of leverage for an entrepreneurial firm is given by the ratio of the market value of debt and private value of the firm, given by the sum of the entrepreneur's private value of equity and the market value of debt. We dub this ratio "private leverage" to highlight the impact of entrepreneurial risk aversion and non-diversifiable idiosyncratic risk exposure on the entrepreneurial firm's leverage decision. Our model includes the standard complete-markets (contingent claim) trade-off model of capital structure (i.e. Leland (1994)) as a special case. Note that this leverage ratio is forward looking and incorporates the effect of future external equity financing and leverage adjustment on current decision making, among other various margins.

Our main findings are as follows. First, diversification benefits of debt are large. Even when we turn off the tax benefits of debt, the entrepreneurial firm still issues a significant amount of outside debt for diversification benefits. The more risk-averse the entrepreneur is, the more he values the diversification benefit of risky debt, hence the higher his private leverage. At the first glance, this result might appear counterintuitive: more risk-averse entrepreneurs ought to be less aggressive in terms of financial policies (e.g. a lower leverage). We may reconcile this result as follows. The more risk-averse entrepreneur has a lower subjective valuation of his business project and has a stronger incentive to sell his firm. Since the (diversified) lenders demand no premium for bearing the entrepreneurial firm's idiosyncratic risk, we expect that more risk-averse entrepreneurs sell a higher fraction of their firms via outside debt.

While the effect of risk aversion on leverage is monotonically increasing, there are two opposing effects of risk aversion on debt coupon. On the one hand, the diversification benefits suggests that coupon increases with entrepreneurial risk aversion. On the other hand, the more risk-averse entrepreneur *ex post* defaults earlier for a given level of coupon, lowering the firm's ability to issue

debt. We refer to the latter effect as the distance-to-default effect. The net impact of risk aversion on debt coupon is therefore ambiguous. For example, when the asset recovery rate is high (i.e. when loss given default is low), then the distance-to-default effect is less relevant, which in turn makes debt coupon increasing with risk aversion. However, when loss given default is high, debt coupon may decrease with risk aversion.

Second, the entrepreneur's private value of his equity in our incomplete-markets model behaves differently from the market value of equity held by diversified investors. Inspired by important insights in Black and Cox (1976), Leland (1994) initiated strong interests on structural models of credit risk and capital structure. In a complete-markets credit risk/capital structure model (e.g. Leland (1994)), equity value is convex, which follows from the Black-Scholes-Merton insight that equity is a call option on firm value. Unlike the complete-markets model, our model predicts that the private value of equity (after debt issuance) is not necessarily globally convex in cash flow. Indeed, the entrepreneur's precautionary saving demand makes his private value of equity potentially concave in cash flow when precautionary saving motive and/or idiosyncratic volatility is sufficiently high.

This finding has important implications for the entrepreneur's project choice decisions. Jensen and Meckling (1976) point out that managers of public firms have incentives to invest in excessively risky projects after debt is in place because the convexity feature of equity. In our model, the risk-averse entrepreneur discounts his private valuation of equity because he bears non-diversified idiosyncratic risks. When the degree of risk aversion is high enough, his private value of equity decreases with idiosyncratic volatility of the project. As a result, he may prefer to invest in a low idiosyncratic volatility project, overturning the asset substitution result of Jensen and Meckling (1976). Our model provides one potential explanation for the (weak) empirical and survey evidence on the quantitative importance of asset substitution and risk seeking incentive (see Graham and Harvey (2001) and other related papers).

Third, the entrepreneur demands not only a systematic risk premium but also an idiosyncratic risk premium due to the lack of diversification. We derive an analytical formula for the idiosyncratic risk premium whose key determinants are risk aversion, idiosyncratic volatility and the sensitivity of entrepreneurial value of equity with respect to cash flows. We show that when the entrepreneurial firm is close to default, the systematic risk premium approaches infinity, while the idiosyncratic risk premium is still finite. The idiosyncratic risk premium goes up as the entrepreneurial firm is

close to going public.

Finally, we show that the leverage ratio can drop substantially when the entrepreneur has access to external equity (i.e. the cash-out option) to diversify his project's idiosyncratic risks. Intuitively, the incremental value of debt financing is lower when the entrepreneur substitutes some debt with the expected use of the cash-out option in the future. A higher debt value increases the "strike" price of exercising the cash-out option because debt needs to be called back at par when the firm cashes out. Hence, the distance to cash-out is higher and the present value of the cash-out option is lower *ceteris paribus* (due to debt overhang). The entrepreneur maximizes his *ex ante* private value of the firm by trading off debt issuance/default option against the value of the cash-out option. The more attractive the external equity financing (cash-out option) is, the lower the firm's private leverage and debt coupon are. Intuitively, the more attractive the entrepreneur's alternative financing instrument (external equity in our example) is, the less aggressive the current financial policy (external debt) needs to be.

We now turn to the related literature. We provide a generalized tradeoff model for the entrepreneurial firm's capital structure under incomplete markets, where the risky outside debt offers an additional diversification benefit over inside equity. Our model includes the structural (complete-markets) credit risk/capital structure models such as the workhorse Leland (1994) model as a special case. We show that precautionary saving demand plays an important role in determining the entrepreneurial firm's leverage and default strategies.

Our model relates to the incomplete-markets consumption smoothing/precautionary saving literature.³ For analytical tractability reasons, we adopt the expected constant-absolute-risk-averse (CARA) utility specification as Merton (1971), Caballero (1991), Kimball and Mankiw (1989), and Wang (2006). Our model contributes to this literature by extending the CARA-utility-based precautionary saving problem to allow the entrepreneur to diversify his idiosyncratic risks via exit strategies such as cash-out and default.

Our model also links to the dynamic portfolio choice/hedging literature (Merton (1971, 1973)). Unlike the standard portfolio choice literature, our model predicts an interdependence between portfolio choice/hedging demand and the entrepreneur's optimal default and leverage policies. The model also contributes to the option exercising/valuation problem under incomplete markets. Miao

³Hall (1978) pioneered the Euler equation approach to analyze intertemporal consumption decision and showed that consumption is a martingale (under certainty equivalence (i.e. quadratic utility)). See Deaton (1992) and Attanasio (1999) for recent surveys.

and Wang (2007) analyze the impact of the entrepreneur's non-diversifiable idiosyncratic risk on his growth option exercising decision. This paper focuses on the entrepreneurial firm's investment and financing (internal versus external, debt versus equity), and endogenous default and cash-out decisions.

Our paper builds on some recent work on entrepreneurial finance, particularly Heaton and Lucas (2004), which emphasizes the diversification benefit of risky debt and studies the impact of non-diversifiable idiosyncratic risks on the entrepreneur's financing and investment decisions. One key difference is that our model is dynamic and their model is static. The key driving force in our model is the entrepreneur's intertemporal consumption smoothing/precautionary saving motive. In their static model, consumption is equal to terminal wealth and hence risk aversion plays the key role. Our dynamic framework models the endogenous default/cash-out decisions as a perpetual American option exercising problem under incomplete markets. Other differences between the two papers include: We treat both taxes on entrepreneurial business income/capital gains, and taxes on public firms, while they do not. We parameterize the cost of financial distress as in Leland (1994) and other tradeoff models, while they use adverse selection as the cost of external financing. We allow the entrepreneur to invest in the risky asset to partially hedge his project risk and hence include the complete-markets setting (Leland (1994)) as a special case, while they assume away hedgable component of risk. Finally, we use CARA utility, while they use constant relative risk aversion (CRRA) utility.

The remainder of the paper is organized as follows. Section 2 describes the entrepreneur's interdependent dynamic decision problem. Section 3 uses the backward induction to solve the entrepreneur's consumption, saving, portfolio choice and the firm's capital structure and cash-out/default decisions. Section 4 analyzes the effect of risky debt for the entrepreneur's financing decisions. Section 5 studies the effect of cash-out on entrepreneurial financing and firm value as an additional diversification channel. Section 6 considers the entrepreneur's endogenous project choice after debt is in place. Section 7 concludes. Technical details are relegated to appendices.

2 Model Setup

Consider an infinitely-lived risk-averse entrepreneur's decision problem in a continuous-time setting. The entrepreneur derives utility from a consumption process $\{c_t : t \geq 0\}$ according to the following

time-additive utility function:

$$\mathbb{E} \left[\int_0^{\infty} e^{-\delta t} u(c_t) dt \right], \quad (1)$$

where $\delta > 0$ is the entrepreneur's subjective discount rate and $u(\cdot)$ is an increasing and concave function. For analytical tractability, we adopt the CARA utility.⁴ That is, let $u(c) = -e^{-\gamma c}/\gamma$, where $\gamma > 0$ is coefficient of absolute risk aversion, which also measures precautionary motive.⁵

The entrepreneur has standard financial investment opportunities (as in Merton (1971)), in that he allocates his liquid financial wealth between a risk-free asset which pays a constant rate of interest r and a market portfolio (the risky asset). Let P_t denote the price of the market portfolio and assume that its return is independently and identically distributed, in that

$$dP_t/P_t = \mu_p dt + \sigma_p dB_t, \quad (2)$$

where μ_p is the expected return on the risky asset and σ_p is the return volatility. Let

$$\eta = \frac{\mu_p - r}{\sigma_p} \quad (3)$$

denote the after-tax Sharpe ratio of the market portfolio.

The entrepreneur has a take-it-or-leave-it project at time zero. If he chooses to start the project, he needs to pay a one-time fixed cost I at time 0. We assume that the entrepreneur sets up a separate entity such as limited liability company (LLC) to run the project. The LLC structure allows the entrepreneur to face a single-layer taxation for his business income and also achieves limited liability.⁶ The entrepreneurial firm pays taxes for both his (flow) business profit and also capital gains when the entrepreneur sells his business. Let τ_e and τ_g denote the respective tax rates for the flow business profit and capital gains upon sale.

After setting up the firm and starting the project, the entrepreneur collects the project's stochastic revenue at the rate y_t and incurs the (flow) operating cost at the constant rate of w , provided

⁴The CARA utility specification proves tractable in incomplete markets consumption-saving problems. Kimball and Mankiw (1989), Caballero (1991), Svensson and Werner (1993), and Wang (2006) have all adopted this utility specification in various precautionary saving models. Miao and Wang (2007) use this utility specification to analyze a real option exercising problem when the decision maker faces uninsurable idiosyncratic risk from his investment opportunity.

⁵Precautionary saving is motivated by the consumption Euler equation analysis to focus on uncertainty of income rather than on risk attitude towards wealth and consumption. Leland (1968) is among the earliest studies on precautionary saving models. Kimball (1990) links the degree of precautionary saving to the convexity of the marginal utility function $u'(c)$. By drawing an analogy to the theory of risk aversion, Kimball (1990) defines $-u'''(c)/u''(c)$ as the coefficient of absolute prudence. For CARA utility, we have $-u'''(c)/u''(c) = \gamma$.

⁶Another form of the firm's legal entity with both limited liability and the single-layer taxation is the S corporation.

that the project is not liquidated. That is, the project's time- t (flow) operating profit is given by $(y_t - w)$. The operating cost w generates operating leverage. Assume that the stochastic revenue process $\{y_t : t \geq 0\}$ follows a geometric Brownian motion (GBM) given by:

$$dy_t = \mu y_t dt + \omega y_t dB_t + \epsilon y_t dZ_t, \quad y_0 \text{ given}, \quad (4)$$

where μ is the expected growth rate of the revenue, ω and ϵ are the corresponding volatility parameters, and B and Z are independent standard Brownian motions driving the (market) systematic and idiosyncratic risks, respectively. Intuitively, we may interpret $\omega > 0$ and $\epsilon \geq 0$ as systematic and idiosyncratic volatility parameters of the revenue growth. The project's total revenue growth volatility σ is then given by

$$\sigma = \sqrt{\omega^2 + \epsilon^2}.$$

As we will show, these different volatility parameters ω , ϵ , and σ have different effects on the entrepreneurial decision making. The entrepreneur can dynamically trade the risky asset to hedge against his business risk. In general, dynamic trading in our entrepreneurial setting does not necessarily complete the market unlike the standard Black-Scholes-Merton (option pricing) paradigm. That is, when the revenue process (4) and the market portfolio return (2) are not perfectly correlated (i.e. $\epsilon > 0$), the entrepreneur bears non-diversifiable idiosyncratic risks from owning the project. This non-diversifiable risk has significant impact on the entrepreneurial decision making and invalidates the standard valuation and capital structure analysis.

The entrepreneur finances the initial fixed cost I of investment via his own funds (internal financing) and external financing (external debt and equity). The natural interpretation of external debt is bank loan. The entrepreneur uses the firm's assets as collateral to borrow. That is, debt is secured. The limited-liability feature of LLC implies that debt is also non-recourse. In addition to bank debt, the entrepreneur also has access to external equity markets either via public securities markets or direct equity sale to (diversified investors). Accessing to external equity markets involves fixed costs such as transaction costs for initial public offering (IPO) or brokerage fees, which particularly discourage smaller firms from using external equity.⁷ Indeed, empirically, entrepreneurial firms mostly use bank debt as the primary form of external financing, particularly for smaller ones.⁸

⁷While we do not formally model dilution costs of information-sensitive claims such as (external equity) due to adverse selection (Myers and Majluf (1984)), we may potentially link and relate the fixed costs to these frictions.

⁸See Vissing-Jorgensen and Moskowitz (2001), Gentry and Hubbard (2004), and Heaton and Lucas (2004), among others.

We abstract away from other frictions such as borrowing constraints that the entrepreneur may face. Introducing financial constraints does not change the key economic mechanism of our model: the effect of the non-diversifiable risk on entrepreneurial financing decisions.

Provided that the fixed cost of accessing external equity K is sufficiently large, the entrepreneur rationally defers equity financing into the future for the standard option value argument. Therefore, at time 0, the model predicts that the entrepreneur uses external debt to reduce his exposure and partially to diversify his idiosyncratic project risk (as in Heaton and Lucas (2004)). This diversification benefit argument does not apply to firms held by owners who have diversified portfolios as in standard law-of-one-price-based valuation theory.⁹ Note that diversification benefits for the entrepreneur exist only when the debt is risky.

We assume that debt is issued at par and is an interest-only debt for analytical tractability reasons as in Leland (1994) and Duffie and Lando (2001).¹⁰ Let b denote the coupon payment of debt and F_0 denote the par value of debt. The remaining cash flow from operation after debt service and tax payments (i.e. $(1 - \tau_e)(y - w - b)$) accrues to the entrepreneur. The terms of debt issuance are determined at time 0. After debt is in place, at each point in time $t > 0$, the entrepreneur continues his project until he decides either to default on his outstanding debt which will lead to the liquidation of his firm, or to cash out by selling his firm to a diversified buyer by paying the fixed cash-out transaction cost and triggering the capital gains taxes. Both default and cash-out decisions are one-time irreversible decisions. Default and cash-out resemble American-style put and call options on the underlying non-tradeable entrepreneurial firm. By American style, we mean that the entrepreneur can endogenously time his default and cash-out decisions, both of which depend on the firm's operating performance. Neither default nor cash-out options is tradeable and cannot be evaluated by the standard dynamic replication argument (Black-Scholes-Merton) because the entrepreneurial firm's risk is not spanned by the market portfolio. The entrepreneur chooses the default and the cash-out timing policies to maximize his own utility after he chooses time-0 debt level for the firm. The entrepreneur's default and cash-out timing strategies are not contractible at time 0. There is an inevitable conflict of interest between financiers and the entrepreneur (e.g.

⁹The standard finance valuation exercise starts with an exogenously specified cash flow project and a cost of capital calculation using risk-return models such as the capital asset pricing model (CAPM). Only systematic risk enters the calculation of the cost of capital and valuation as in the CAPM.

¹⁰We may extend our model specification to allow for debt with shorter maturity. This complicates the analysis without changing the key economic intuition of our paper: the diversification benefits of risky debt. However, the quantitative implications are potentially different.

Jensen and Meckling (1976)).

If the entrepreneur defaults on the firm's debt at his endogenously chosen (stochastic) time T_d , the firm goes into bankruptcy. After bankruptcy, the lender takes control of the firm and liquidates the firm or sells the firm to the potential investors. Bankruptcy *ex post* is costly as in standard tradeoff models of capital structure. Assume that the liquidation/sale value of the firm is equal to a fraction α of an all equity-value (i.e. unlevered) public firm.¹¹ Let $A(y)$ denote the after-tax unlevered market value of the firm. The remaining fraction $(1 - \alpha)$ accounts for bankruptcy costs.¹² While there is potential room for the lender and the entrepreneur to renegotiate *ex post*, we abstract from this issue given the focus of our paper.¹³

Intuitively, the entrepreneur defaults when the firm does sufficiently poorly to walk away from his liability. Assume that absolute priority is enforced. The limited liability and the entrepreneur's voluntary default imply that the entrepreneur receives nothing upon default and the lender collects the proceeds from liquidation. After liquidation, the entrepreneur is no longer exposed to the firm's idiosyncratic risk. In our generalized tradeoff model, the entrepreneur trades off diversification benefits against the default and agency costs of debt. Unlike for the public firm, the tax implications for the entrepreneurial firm is less obvious due to the single-layer taxation feature of the entrepreneurial firm.

When the firm does sufficiently well, the entrepreneur may find it optimal to cash out by incurring the fixed cash-out cost K . The entrepreneur then triggers capital gains taxes. As we will show below, capital gains taxes have potentially important effects on the firm's cash-out timing decision.¹⁴ As in the real world, the entrepreneur needs to retire the firm's debt obligation at par F_0 in order to cash out in our model. The gross proceeds that the entrepreneur obtains from selling his firm is fairly priced by the competitive capital market. Since new owners are well diversified and do not demand the idiosyncratic risk premium, we will use the complete-markets solution to obtain the sale value of the firm. The new owners will optimally relever the firm by issuing a

¹¹This assumption is not crucial to the results of our model. See Mello and Parsons (1992).

¹²We also assume that the tax shields are lost after default. This simplifying assumption does not change results in any fundamental way and can be easily relaxed.

¹³Extending the model to allow for richer specifications and predictions when the firm is experiencing difficult times does not change the model's predictions in any significant way with respect to the effect of non-diversifiable risk on entrepreneurial financing.

¹⁴Our model can be extended to account for other features such as depreciation, which may further complicate the tax treatment. For simplicity, we abstract away from these issues, which are not crucial to the economic mechanism of this paper.

perpetual debt with a different coupon level as in the complete-markets model of Leland (1994). Let τ_m denote the effective marginal tax rate for the public firm after the entrepreneur sells his firm. Unlike the entrepreneurial firm, the public firm is potentially subject to double taxation (at the corporation and the individual’s levels) and hence we will interpret τ_m accordingly (Miller (1977)). The entrepreneur and the market have rational expectations and hence the entrepreneur also benefits from this releverage upon the firm’s exit. To summarize, at the endogenously chosen stochastic cash-out time T_u , the entrepreneur pays the fixed transaction cost K , retires debt at par F_0 , and pays the capital gains taxes to capitalize on the value of the project.

After the entrepreneur exits from his business by either defaulting on his debt or cashing out his business, he retires and has no other non-financial income. He then solves a standard complete-markets consumption and portfolio choice problem as in Merton (1971). Extending our model to allow for sequential rounds of entrepreneurial activities will complicate our analysis. We leave this extension for future research.

We finally close the model by allowing the entrepreneur to have a one-time irreversible choice of his project’s idiosyncratic volatility ϵ (immediately) after he borrows F_0 from the bank. The bank has rational expectations and anticipates the entrepreneur’s *ex post* choice of the project’s ϵ and hence charges the corresponding credit spread *ex ante* to make zero economic profit. The time inconsistency nature of the entrepreneur’s project choice induces interesting predictions on entrepreneurial decision making, idiosyncratic risk premium, and debt pricing. This particular heterogeneity across projects allows us to analyze the standard risk shifting/asset substitution incentive (Jensen and Meckling (1976)) for non-diversified risk-averse decision makers such as entrepreneurs. Our model provides a potential reconciliation with the empirical findings that risk-shifting issue is quantitatively a second-order issue.

3 Model Solution

We solve the entrepreneur’s optimization problem as follows. Section 3.1 summarizes the complete-markets solution for firm value and financing decisions if the firm is owned by well diversified investors. This complete-markets solution (Leland (1994)) gives the “cash-out” value for the entrepreneur from selling his firm and also serves as a benchmark for our comparison analysis. In Section 3.2, we analyze the the entrepreneur’s interdependent consumption/saving, portfolio choice,

cash-out/default, and initial investment and financing decisions.

3.1 Complete-markets firm valuation and financing policy

After the entrepreneur cashes out his equity by selling his firm to well diversified investors, the new owners reliver the firm by issuing a perpetual debt with a new coupon level b to maximize *ex ante* firm value. The new owners trade off the tax benefits of debt against bankruptcy and agency costs. For analytical convenience, we assume that there is no re-adjustment of capital structure in the future (Leland (1994)).¹⁵ See Appendix A for details for the complete-markets analysis, which essentially follows from Leland (1994), Goldstein, Ju, and Leland (2001), and Miao (2005). Here we highlight a few pricing formulae that we use in the main body of the text.

First, consider valuation and risk premium without financial leverage. As in the standard CAPM model, only the systematic risk demands a risk premium under the complete-markets setting. In our dynamic setting, the systematic risk premium for the revenue component is captured by the wedge between the expected revenue growth rate μ and the risk-adjusted expected growth rate of revenue ν given below:

$$\nu = \mu - \omega\eta. \quad (5)$$

In addition to the standard risk premium effect, the after-tax value of unlevered public firm $A(y)$ also has an embedded abandonment option (induced by the flow operating cost w). The after-tax unlevered firm value $A(y)$ given by the following analytical formula captures both the risk premium and abandonment option value:

$$A(y) = (1 - \tau_m) \left[\left(\frac{y}{r - \nu} - \frac{w}{r} \right) - \left(\frac{y_a}{r - \nu} - \frac{w}{r} \right) \left(\frac{y}{y_a} \right)^{\theta_1} \right], \quad (6)$$

where the abandonment threshold y_a is given in (A.7), and θ_1 is a parameter given by (A.8). Recall that the effective tax rate for the public firm is τ_m . This tax rate takes into account the double taxation for the public firm (Miller (1977)).

Now turn to valuation for a levered public firm. Given coupon rate b and default threshold y_d ,

¹⁵We abstract away from the dynamic capital structure decisions after the entrepreneur cashes out to keep the analysis tractable and also analogous to our treatment before the entrepreneur exits. While extending the model by allowing for dynamic financing adjustments will enrich the model, it complicates our analysis without changing the key economic tradeoff that we focus: the impact of idiosyncratic risk on entrepreneurial financing decisions. We leave extensions along the line of Goldstein, Ju, and Leland (2001) for future research.

the market value of equity $E(y; y_d)$ and the levered firm value $V(y; y_d)$ are respectively given by

$$E(y; y_d) = (1 - \tau_m) \left[\left(\frac{y}{r - \nu} - \frac{w + b}{r} \right) - \left(\frac{y_d}{r - \nu} - \frac{w + b}{r} \right) \left(\frac{y}{y_d} \right)^{\theta_1} \right], \quad (7)$$

$$V(y; y_d) = A(y) + \frac{\tau_m b}{r} \left[1 - \left(\frac{y}{y_d} \right)^{\theta_1} \right] - (1 - \alpha) A(y_d) \left(\frac{y}{y_d} \right)^{\theta_1}, \quad (8)$$

Equation (7) shows that equity value is equal to the after-tax present value of profit flows minus the present value of the perpetual coupon payments plus an option value to default. The term $(y/y_d)^{\theta_1}$ may be interpreted as the “risk-adjusted” Arrow-Debreu price of a unit of claim contingent on the event of default. Equation (8) shows that the levered market value of the firm is equal to the after-tax unlevered firm value plus the present value of tax shields minus bankruptcy costs.

After debt is in place, there is a conflict between equityholders and the debtholders. Equityholders choose the default threshold y_d to maximize equity value $E(y; y_d)$. Appendix A shows that the default boundary is given by

$$y_d^* = \frac{r - \nu}{r} \frac{\theta_1}{\theta_1 - 1} (b + w). \quad (9)$$

While y_d^* is chosen to maximize $E(y)$, coupon b is chosen to maximize *ex ante* firm value $V(y)$. Let b^* denote the firm-value-maximizing coupon, in that $b^* = \arg \max V(y; y_d^*)$, and $V^*(y)$ denote the corresponding levered market value of the firm as a function of the (initial) cash flow y , in that

$$V^*(y) = \max_b V(y; y_d^*). \quad (10)$$

3.2 Entrepreneur’s decision making

We solve the entrepreneur’s decision making in three steps by backward induction. First, we briefly summarize the entrepreneur’s consumption/saving and portfolio choice problem after he retires from his business either via cashing out or defaulting on debt. This optimization problem is the same as in Merton (1971), a (dynamic) complete-markets consumption/portfolio choice problem. Second, we solve the entrepreneur’s joint consumption/saving, portfolio choice, default, and cash-out decisions after debt is in place when the entrepreneur runs the firm. Finally, we solve the entrepreneur’s initial investment and financing decision at time 0.

Let $\{x_t : t \geq 0\}$ denote the entrepreneur’s financial wealth process and let ϕ_t denote the amount of his financial wealth invested in the market portfolio at time t . The entrepreneur’s initial financial

wealth x_0 after financing his business is equal to his endowment \underline{x} plus debt proceeds F_0 and minus the investment cost I , in that

$$x_0 = \underline{x} - (I - F_0). \quad (11)$$

Consumption/saving and portfolio choice after exit. After he exits from his business (via either default or cash-out), the entrepreneur no longer has business income and lives on his financial wealth. The entrepreneur's optimization problem becomes the standard complete-market consumption and portfolio choice problem. By Merton (1971), the entrepreneur's value function $J^e(x)$ is given by the following explicit form:

$$J^e(x) = -\frac{1}{\gamma r} \exp \left[-\gamma r \left(x + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2} \right) \right]. \quad (12)$$

Entrepreneur's decision making while running the firm. We now consider the decision problem when the entrepreneur runs his firm. The entrepreneur receives his non-tradeable business income, pays operating cost, services the debt and pays taxes until he exits via either default or cash-out. Let $J^s(x, y)$ denote his value function. In Appendix B, we show that the value function takes the following explicit exponential form:

$$J^s(x, y) = -\frac{1}{\gamma r} \exp \left[-\gamma r \left(x + G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2} \right) \right], \quad (13)$$

where $G(y)$ is given in Theorem 1 below. As shown in Miao and Wang (2007), $G(y)$ is the entrepreneur's certainty equivalent wealth from his ownership of the firm. We will refer to $G(y)$ as the entrepreneur's private value of equity.

At the moment of default and cash-out, the value functions $J^s(x, y)$ and $J^d(x)$ must satisfy certain value-matching and smooth-pasting conditions described in Appendix B. These conditions determine a default boundary $y_d(x)$ and a cash-out boundary $y_u(x)$. In general these boundaries depend on the wealth level x . Note that equation (13) implies that $G(y)$ is additively separable from financial wealth x in $J^s(x, y)$. This absence of wealth effect implies that the default and cash-out boundaries $y_d(x)$ and $y_u(x)$ are independent of wealth. We thus simply use y_d and y_u to denote the default and cash-out thresholds, respectively.

While CARA utility does not capture the wealth effect, we emphasize that the main results and insights of our paper (the effect of non-diversifiable idiosyncratic shocks on investment timing) do not rely on the particular choice of this utility function. As we show below, the driving force of

our results is the precautionary savings effect, which is captured by utility functions with convex marginal utility such as CARA.¹⁶ While power utility is more commonly used and more appealing for quantitative analysis, this utility specification will substantially complicate our analysis since it will lead to a much harder two-dimensional double-barrier free-boundary problem.

We summarize the solution for consumption/saving, portfolio choice, default trigger y_d , and cash-out trigger y_u in the following:

Theorem 1 *The entrepreneur exits from his business when the cash flow process $\{y_t : t \geq 0\}$ reaches either the default threshold y_d or the cash-out threshold y_u , whichever occurring the first. When the entrepreneur runs his firm, he chooses his consumption and portfolio rules as follows:*

$$\bar{c}(x, y) = r \left(x + G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2} \right), \quad (14)$$

$$\bar{\phi}(x, y) = \frac{\eta}{\gamma r \sigma_p} - \frac{\omega}{\sigma_p} y G'(y), \quad (15)$$

where $(G(\cdot), y_d, y_u)$ solves the free boundary problem given by the differential equation:

$$rG(y) = (1 - \tau_e)(y - b - w) + \nu y G'(y) + \frac{\sigma^2 y^2}{2} G''(y) - \frac{\gamma r \epsilon^2 y^2}{2} G'(y)^2, \quad (16)$$

subject to the following (free) boundary conditions

$$G(y_d) = 0, \quad (17)$$

$$G'(y_d) = 0, \quad (18)$$

$$G(y_u) = V^*(y_u) - F_0 - K - \tau_g(V^*(y_u) - K - I), \quad (19)$$

$$G'(y_u) = (1 - \tau_g)V^{*'}(y_u), \quad (20)$$

where $V^*(y)$ is defined in (10) and F_0 is the par value of the perpetual debt.

The differential equation (16) provides a certainty equivalent valuation for $G(y)$ from the entrepreneur's perspective. The last (nonlinear) term captures the intuition behind the discount due to the non-diversifiable idiosyncratic risk. Intuitively, a higher risk aversion parameter γ , a larger

¹⁶A well-known implication of CARA-utility-based models is that consumption and wealth may sometimes turn negative (see e.g. Merton (1971), Grossman (1976) and Wang (1993)). Cox and Huang (1989) provide analytical formulae for consumption under complete markets for CARA utility with non-negativity constraints. However, in our incomplete-markets setting, imposing non-negativity constraints substantially complicates the analysis. Intuitively, requiring consumption to be positive increases the entrepreneur's demand for precautionary saving because he will increase his saving today to avoid hitting the constraints in the future. The induced stronger precautionary saving demand in turn makes our results (such as diversification benefits of outside risky debt) stronger.

discount on $G(y)$ due to the non-diversifiable idiosyncratic risk. The next section provides more detailed analysis on the impact of idiosyncratic risk on the entrepreneur's subjective valuation $G(y)$.

Equation (17) comes from the value-matching condition for the entrepreneur's default decision. It states that the private value of equity upon default is equal to zero. Equation (18) follows from the smooth-pasting condition. It can be interpreted as the optimality condition from the maximization of the private value of equity.

Now we turn to the cash-out boundary. Because the entrepreneur pays the fixed cost K and triggers capital gains when cashing out, he naturally has incentives to wait before cashing out. However, waiting to cash out reduces his diversification benefits *ceteris paribus*. The entrepreneur optimally trades off along these margins when choosing the timing of cashing out.

Recall that $V^*(y_{T_u})$ gives the exit value of the entrepreneurial firm when cashing out at time T_u . The capital gains taxes upon cashing out is $\tau_g(V^*(y_{T_u}) - F_0 - K - (I - F_0))$. The entrepreneur's financial wealth changes discretely from x_{T_u-} just prior to cashing out to $x_{T_u} = x_{T_u-} + V^*(y_{T_u}) - F_0 - K - \tau_g(V^*(y_{T_u}) - K - I)$ immediately after. The value-matching condition (19) at the cash-out boundary states that the private value of equity upon the firm's cashing out is equal to the after-tax value of the public firm value after the entrepreneur pays the fixed costs K , retires outstanding debt at par F_0 , and pays capital gains taxes (equivalently, reflects the entrepreneur's wealth change from x_{T_u-} to x_{T_u}). The smooth-pasting condition (20) can be viewed as the optimality condition for the cash-out decision.

Initial financing and investment decision. Theorem 1 characterizes the entrepreneur's decisions after debt is in place. We now complete the model solution by endogenizing the entrepreneur's initial investment and financing decision. Note that the entrepreneur's initial wealth x_0 immediately after investment and financing is given by (11). At time zero, the entrepreneur chooses a coupon rate b to solve the following problem:

$$\max_b J^s(\underline{x} + F_0 - I, y_0), \quad (21)$$

subject to the requirement that debt be issued at par, i.e. $F_0 = F(y_0)$, where $F(y)$ denotes the market value of debt. In Appendix C, we provide an explicit formula for $F(y)$.

At time zero, the entrepreneur launches the project if his indirect utility from the project is

higher than the indirect utility without the project, i.e.,

$$\max_b J^s(\underline{x} + F_0 - I, y_0) > J^e(\underline{x}). \quad (22)$$

Since equity is held by the entrepreneur and is not tradable, we only have private valuation $G(y)$. Debt is issued to diversified investors and is priced at market value $F(y)$. Intuitively, we define the private value of the entrepreneurial firm, denoted by $S(y)$, as follows:

$$S(y) = G(y) + F(y). \quad (23)$$

We may interpret $S(y)$ as the fair price that an investor needs to pay in order to acquire the entrepreneurial firm by paying $G(y)$ to the entrepreneur and $F(y)$ to the lender.¹⁷

Since the entrepreneur receives $F(y_0)$ at time 0 in exchange for future coupon payments, the private firm value for the entrepreneur at time 0 is $S(y_0) = G(y_0) + F(y_0)$. Using the exponential form (13) for the entrepreneur's value function $J^s(x, y)$, it is straightforward to show that the maximization problem stated in (21) is equivalent to the following one:

$$\max_b S(y_0) = G(y_0) + F(y_0). \quad (24)$$

Note the conflicts of interest between the entrepreneur and the lender. When choosing the debt coupon, the entrepreneur maximizes his private value of the firm, $S(y_0)$, because he internalizes the benefits and costs of debt issuance. After debt is in place, the entrepreneur chooses default and cash-out thresholds to maximize his private value of equity $G(y)$. Unlike publicly held firms, the optimal coupon in our setting maximizes the private value $S(y_0)$, but not the market value $V(y_0) = E(y_0) + F(y_0)$, which is the conventional way to calculate firm value as in Leland (1994) or any other structural default/credit risk models.

We may interpret our model's implication on capital structure as a generalized tradeoff model of capital structure for the entrepreneurial firm, where the entrepreneur trades off the benefits of debt (tax shields and diversification) against the costs of debt (bankruptcy/inefficient liquidation and agency conflicts between the entrepreneur and outside lenders). Similar to the classic tradeoff model, the objective function in our model for debt choice is private value of firm $S(y)$, the sum of private value of equity $G(y)$ and public value of firm $F(y)$. The natural measure of leverage (based

¹⁷Assume that the entrepreneur and the lender do not receive any surplus from the takeover/restructuring. Hence, the entrepreneur and the lender demands $G(y)$ and $F(y)$, respectively.

on the optimization problem) is the ratio between private value of equity $G(y)$ and the private value of firm $S(y)$, in that

$$L(y) = \frac{G(y)}{G(y) + F(y)} = \frac{G(y)}{S(y)}, \quad (25)$$

We dub $L(y)$ as private leverage to reflect the impact of idiosyncratic risk on the leverage choice.

Given the exponential forms (13) and (12) for the entrepreneur's value functions, we can also deduce from (22) that the entrepreneur invests in the project if the private value of the firm is larger than the investment cost, $\max_b S(y_0) > I$. That is, the entrepreneur follows a modified net present value rule in that he computes the net present value of the project using his private valuation, instead of market valuation.

Special cases: Immediate exits via default or cash-out. There are two special cases. The first is the one where the cost of cashing out is sufficiently small so that it is optimal for the entrepreneur to sell the firm immediately to diversified investors. That is, the cash-out option is immediately worth exercising at time 0 ($y_u < y_0$). The other special case is the one where asset recovery rate is sufficiently high or the entrepreneur is sufficiently risk averse. Then, the entrepreneur raises as much debt as possible (i.e. 100% sale to the lender) and then defaults immediately ($y_d > y_0$). In our analysis below, we will consider parameter values that rule out these two special cases.

We next turn to the model's predictions and results. We proceed in three steps. In Section 4, we analyze the case where the entrepreneur only has the default option. This setting allows us to focus on the economics of non-diversifiable idiosyncratic risk on the entrepreneur's default and financing decisions. In Section 5, the entrepreneur can exit his business via either default or cash-out, allowing the entrepreneur to diversify his idiosyncratic risks. In Section 6, we endogenize the entrepreneur's choice of the project's idiosyncratic risk. We show that the risk averse entrepreneur optimally trades off his risk-sharing (diversification) benefits with the option-payoff induced risk seeking incentive.

4 Risky debt, endogenous default, and diversification

In this section, we focus on the analysis of the default option and diversification benefit of risky debt by temporarily shutting down the cash-out option. This setting is a special case of the optimization

problem characterized in Theorem 1, when the cash-out cost is infinitely large.¹⁸

To solve the model numerically, we use the following (annualized) baseline parameter values: the risk-free interest rate $r = 0.03$, the expected growth rate of revenue $\mu = 0.04$, systematic volatility of growth rate $\omega = 0.1$, idiosyncratic volatility $\varepsilon = 0.2$, and the market price of risk $\eta = 0.4$. We set the asset recovery rate $\alpha = 0.6$. We set the effective marginal (Miller) tax rate τ_m to 11.29% as in Graham (2000) and Hackbarth, Hennessy and Leland (2007).¹⁹ In our baseline parametrization, we set $\tau_e = 0$, which reflects the fact that the entrepreneur can avoid taxes on his business income completely. We focus on $\tau_e = 0$ in order to focus on the role of the diversification benefits of debt by abstracting from tax considerations. But we also consider the case with $\tau_e = \tau_m$ so that we can compare our entrepreneurial model with the complete-markets model of Section 3.1. We consider four values of the risk aversion parameter $\gamma \in \{0, 1, 2, 4\}$. In the baseline model, we also set the operating cost $w = 0$, which simplifies our numerical solution. In Section 5.3, we analyze the case with positive operating cost $w > 0$ and study the role of operating leverage. Finally, we set the initial level of revenue $y_0 = 1$.

4.1 Private value of equity and default threshold

Figure 1 plots private value of equity $G(y)$ and its derivative $G'(y)$ as functions of y . The top two panels are for $\tau_e = 0$ and the bottom two panels are for $\tau_e = \tau_m$. Each panel plots the solutions for three different values of $\gamma = 0, 1, 2$. The limiting solution when $\gamma \rightarrow 0$ corresponds to the complete markets case (Leland (1994)).

When $\tau_e = 0$, the entrepreneur with very low risk aversion ($\gamma \rightarrow 0$) effectively does not issue any debt, because there is neither tax benefits ($\tau_e = 0$) nor diversification benefits ($\gamma \rightarrow 0$). As a result, equity value, which is equivalent to firm value in this case, is the present discounted value of future cash flows (the straight line shown in the top-left panel). By contrast, a risk-averse entrepreneur has incentives to issue debt in order to diversify idiosyncratic risk even when $\tau_e = 0$. Given the entrepreneur's limited liability, he may default at the endogenously chosen default threshold. For $\gamma > 0$, the default threshold y_d is the point at which $G(y)$ touches the horizontal axis. At this

¹⁸In this special case, there is only one endogenous (lower) default boundary. The upper boundary is replaced by the transversality condition: $\lim_{T \rightarrow \infty} E [e^{-\delta T} J^s(w_T, y_T)] = 0$.

¹⁹We may interpret τ_m as the effective Miller tax rate which integrates the corporate income tax, individual's equity and interest income tax. Using the Miller's formula for the effective tax rate, and setting the interest income tax at 0.30, corporate income tax at 0.31, and the individual's long-term equity (distribution) tax at 0.10, we obtain an effective tax rate of 11.29%.

point, $G(y_d) = G'(y_d) = 0$. That is, the private value of equity at the default threshold satisfies the value-matching and smooth-pasting conditions, as in the complete markets case. When $\tau_e = \tau_m$, both the entrepreneurial and public firm will issue debt to take advantage of tax benefits. The bottom two panels of Figure 1 plot this case.

[Insert Figure 1 Here.]

Like the option Delta in financial derivatives literature,²⁰ $G'(y)$ measures the sensitivity of private value of equity $G(y)$ with respect to revenue y . As expected, private value of equity $G(y)$ increases with revenue y , i.e. $G'(y) > 0$. Analogous to Black-Scholes-Merton's observation that firm equity is a call option on firm assets, the entrepreneur's private equity $G(y)$ also has a call option feature. For example, in the complete markets case, equity value is convex in revenue y , reflecting its call option feature.

Unlike the standard Black-Scholes-Merton paradigm, neither the entrepreneurial equity nor the firm is tradable. When the risk averse entrepreneur cannot fully diversify his project's idiosyncratic risk, the global convexity of $G(y)$ no longer holds, as shown in Figure 1 for cases where $\gamma > 0$. The entrepreneur now has precautionary saving demand to partially buffer the project's non-diversifiable idiosyncratic shocks. This precautionary saving effect is greater, when the idiosyncratic volatility ϵy is larger (i.e. either ϵ or revenue y is larger). Moreover, the option (convexity) effect is smaller when revenue y is higher (i.e. the default option is further out of the money). The precautionary saving effect dominates the option effect for sufficiently high revenue y , making $G(y)$ concave in y . When y is low, idiosyncratic volatility ϵy is low. Moreover, the default option is closer to be in the money. The convexity effect of the default option dominates the concavity effect of precautionary saving demand, making $G(y)$ convex in y for sufficiently low y . This tradeoff explains the convexity of $G(y)$ for low y and concavity of $G(y)$ for high y , as shown in the upper panels.

Now we turn to the effect of the entrepreneur's risk aversion γ on his subjective valuation and default threshold y_d . A more risk-averse entrepreneur has a stronger incentive to diversify idiosyncratic risks by retaining less of the firm, contributing to a lower private value of equity $G(y)$, as illustrated in Figure 1.

There are two opposing effects of risk aversion on the default threshold y_d . On the one hand, the more risk-averse entrepreneur has stronger incentives to issue debt, which in turn calls for a

²⁰Option Delta measures the sensitivity of option value with respect to the change of the underlying asset value.

larger coupon b and hence a higher default threshold, *ceteris paribus*. On the other hand, the more risk-averse entrepreneur discounts future cash flows more heavily, which makes them exercise the default option earlier given the same level of coupon b . This effect reduces the entrepreneur’s ability to issue outside risky debt, which leads to a lower coupon, *ceteris paribus*. Figure 1 shows that the diversification effect dominates the latter effect, making the default threshold y_d increasing in risk aversion γ . Next, we analyze the impact of non-diversifiable idiosyncratic risks on the entrepreneurial firm’s capital structure.

4.2 Capital structure for entrepreneurial firms

To highlight the role of idiosyncratic risks in a simplest possible way, we consider two scenarios. We first consider the special case where debt has no tax benefits for the entrepreneur (i.e. $\tau_e = 0$), and then incorporate the tax benefits of debt for the entrepreneur into our analysis.

The top panel in Table 1 provides results for the entrepreneurial firm’s capital structure when $\tau_e = 0$. If the entrepreneur is very close to being risk neutral ($\gamma \rightarrow 0$), the model’s prediction is essentially the same as the complete-market benchmark. In this case, the standard tradeoff theory of capital structure implies that the entrepreneurial firm will be purely financed by equity. The risk-neutral entrepreneur values the firm at its market value 33.33.

For $\gamma = 1$, the entrepreneur borrows $F_0 = 8.28$ in market value with coupon $b = 0.31$ from the lender and values his non-tradable equity G_0 at 14.39, giving the private value of the firm $S_0 = 22.68$. Note the substantial 32% drop of S_0 from 33.33 to 22.68. This drop of S_0 is not primarily due to the default risk premium of the risky debt, since the 10-year cumulative default probability is only 0.4% and the implied credit spread on the perpetual debt is 72 basis points. Instead, the significant drop is mainly due to the entrepreneur’s subjective valuation discount of his non-tradable equity position for bearing non-diversifiable idiosyncratic risks.

For entrepreneurial firms, the natural measure of leverage is private leverage L_0 , which is given by the ratio of public debt value F_0 and private value of the firm S_0 . As we have discussed, L_0 captures the entrepreneur’s tradeoff between private value of equity and public value of debt in choosing debt coupon policy. For $\gamma = 1$, the private leverage ratio is about 36.5%.

[Insert Table 1 Here.]

With a higher risk aversion level $\gamma = 2$, the entrepreneur borrows more ($F_0 = 14.66$) with

a higher coupon ($b = 0.68$).²¹ The entrepreneur values his remaining non-tradable equity with a much smaller value $G_0 = 5.89$. The implied private leverage ratio $L_0 = 71.3\%$ is higher than 36.5%, the value for $\gamma = 1$. The more risk-averse entrepreneur takes on more leverage, which is consistent with the diversification benefits argument – the more risk-averse entrepreneur has incentives to sell more of the firm. The high leverage ratio (i.e. 71.3%) gives rise to a significantly higher credit spread (166 basis points over the risk-free rate), and a much higher 10-year cumulative default probability (12.1%), i.e. non-investment-grade debt. Despite the substantially higher default risk and lower equity value, the private value of the firm S_0 for $\gamma = 2$ is 20.55, only about 9% lower than 22.68, the value for $\gamma = 1$. The reason is that the entrepreneur with $\gamma = 2$ takes on more debt, and thus the increase in the market value of debt partially offsets the decrease in private equity value.

Next, we incorporate the effect of tax benefits for the entrepreneur into our generalized tradeoff model of capital structure for entrepreneurial firms. To compare with the complete-markets benchmark, we set $\tau_e = \tau_m = 11.29\%$. Therefore, the only difference between an entrepreneurial firm and a public firm is that the entrepreneur faces non-diversifiable idiosyncratic risks.

The second panel of Table 1 reports the results. The first row in this panel gives the results for the complete-market benchmark. Facing positive corporate tax rates, the public firm has incentives to issue debt, but is also concerned with bankruptcy costs. By the standard tradeoff theory, the public firm optimally issues debt at the market value $F_0 = 9.29$ with coupon $b = 0.35$, which gives 30.9% initial leverage and 0.3% 10-year cumulative default probability.

Similar to the case $\tau_e = 0$, an entrepreneur facing non-diversifiable idiosyncratic risks has incentives to issue more risky debt to diversify these risks. The second panel of Table 1 reveals that the entrepreneur with $\gamma = 1$ borrows an amount 14.85 (with the coupon rate $b = 0.68$), which is higher than the level $b = 0.35$ for the public firm. The private leverage more than doubles to 67.9%. As a result, the entrepreneur faces a higher default probability and the credit spread of his debt is also higher. With $\gamma = 2$, the amount of debt rises to 16.50, private leverage to 81.4%.

4.3 Determinants of capital structure decisions

We conduct two numerical experiments in Table 2 to further demonstrate the important role of idiosyncratic risks in determining the capital structure of entrepreneurial firms. The first and the last rows of this table record results, obtained in Table 1, for the entrepreneurial firm with $\gamma = 2$

²¹This result does not always hold. We will return to this point when we analyze the setting when $\tau = \tau_m\%$.

and for the public firm with $\gamma = 0$, respectively.

[Insert Table 2 here.]

In the first experiment, we suppose that the public firm issues the same amount of debt $b = 0.85$ and defaults at the same threshold $y_d = 0.47$ as the entrepreneurial firm with $\gamma = 2$. Given these values of coupon b and default threshold y_d , we can calculate the implied market value of equity $E_0 = E(y_0; b, y_d)$ and the market value of the firm $V_0 = V^*(y_0; b, y_d)$. The market leverage is given by the ratio between the market value of debt F_0 and the market value of the firm V_0 . Since $E(y; b, y_d) > G(y; b, y_d)$, the imputed market leverage overstates the value of equity for the entrepreneur by ignoring the idiosyncratic risk premium ($E_0 = 11.10$ and $G_0 = 3.77$), thus leading to a leverage ratio 59.8%, substantially lower than the firm's private leverage $L_0 = 81.4\%$. The large difference between the private and market leverage ratios highlights the economic significance of taking idiosyncratic risks into account in order to correctly compute the value of equity.

In the second experiment, we highlight the impact of the entrepreneur's *endogenous* default decision on the leverage ratio. We consider a public firm that has the same technology/environment parameters as the entrepreneurial firm. Moreover, the public firm has the same debt coupon b on the outstanding perpetual debt as the entrepreneurial firm does ($b = 0.85$ in this example). The key difference is that the default threshold y_d is endogenously determined.

The default threshold y_d for the public firm is 0.35, lower than the threshold $y_d = 0.47$ for the entrepreneurial firm. Intuitively, facing the same coupon b , the entrepreneurial firm defaults earlier than the public firm because defaults allows the entrepreneur to avoid the downside non-diversifiable idiosyncratic risk. The implied shorter distance-to-default for the entrepreneurial firm translates into a substantially higher 10-year default probability (22% for the entrepreneurial firm versus 10% for the public firm) and a higher credit spread (213 basis points for the entrepreneurial firm versus 178 basis points for the public firm). Despite the significant difference in the default thresholds, the leverage for the public firm (evaluated at the optimal default threshold $y_d = 0.35$) is 60.5%, only slightly higher than 59.8%, the market leverage for the entrepreneurial firm.

The results of the preceding two experiments suggest the following. When issuing debt with the same amount of coupon payments, endogenous choice of default timing does not have a significant effect on the valuation and the capital structure of the entrepreneurial and public firms. By contrast, the subjective discount for bearing non-diversifiable idiosyncratic risk plays an important role in

computing the value of equity. When both the debt level and default timing are endogenously chosen, the effect of idiosyncratic risk is much stronger, making the risk averse entrepreneur issue more debt. As a result, the leverage ratio for the entrepreneurial firm is substantially higher than that for the public firm (in our example, 60.5% versus 30.9%). While this result is parameter specific, the analysis provides support for our intuition that the entrepreneur’s subjective valuation discount for bearing non-diversifiable idiosyncratic risk is a key determinant of private leverage for the entrepreneurial firm.

5 External equity: an additional channel for diversification

We now turn to a richer and more realistic setting where the entrepreneur can diversify the idiosyncratic risks through both debt and equity. As in Section 3.1, the entrepreneur avoids the downside risk by defaulting if the firm’s (stochastic) revenue y falls sufficiently low. When the firm does well enough, the entrepreneur capitalizes on the upside by selling the firm (issuing outside equity) to diversified investors.

To conduct numerical analysis, we choose baseline parameter values as in Section 4. For new parameters, we set the effective capital gains tax rate $\tau_g = 0.10$, reflecting the tax deferral advantage. We set the initial investment cost $I = 10$, which is about 1/3 of the market value of project cash-flows. We choose the cash-out cost $K = 27$ to generate a 10-year cash-out probability of 20% (with $\gamma = 2$ and $\tau_e = 0$), which is consistent with success rates of venture capital firms in the data. Finally, we consider operating costs $w \in \{0, 0.2, 0.4\}$.

5.1 Effects of cash-out option and capital gains tax

First, consider the complete-market benchmark. When $\tau_m = 0$, there is no tax benefits or diversification benefits, and the cash-out option is worthless. When the public firm has tax benefits ($\tau_m = 11.29\%$), the firm’s cash-out option is essentially an option to adjust the firm’s capital structure. Given our calibrated fixed cost K , we find that the 10-year cash-out probability is almost zero and hence this option value is close to zero for the public firm. Therefore, we expect that the bulk part of the cash-out option value for entrepreneurial firms comes from the diversification benefits, not from the option value of readjusting leverage.

[Insert Table 3 here.]

Next consider an entrepreneur with risk aversion $\gamma = 1$, who faces a tax system with $\tau_e = \tau_g = 0$. Figure 2 plots the private value of equity $G(y)$ and its first derivative $G'(y)$. The function $G(y)$ smoothly touches the horizontal axis and the straight line for $V^*(y)$. The two tangent points give the default and cash-out thresholds, respectively. For sufficiently low values of revenue y , the private value of equity $G(y)$ is increasing and convex because the default option is deep in the money, generating convexity. For sufficiently high values of y , $G(y)$ is also increasing and convex because the cash-out option is deep in the money. For revenue y in the intermediate range, neither default nor cash-out option is deep in the money. In this range, the precautionary saving motive may be large enough to induce concavity. As shown in the right panel of Figure 1, $G'(y)$ first increases for low values of y , then decreases for intermediate values of y , and finally increases for high values of y .

[Insert Figure 2 here.]

The presence of the cash-out option substantially lowers the initial coupon to $b = 0.11$ from $b = 0.31$ for the firm which only has the default option. The private leverage ratio L_0 at issuance is 13.8%, with a credit spread at 37 basis points, compared to the private leverage ratio $L_0 = 36.5\%$ and credit spread 72 basis points when the firm only has the default option. The 10-year default probability is close to zero, but the 10-year cash-out probability is 12.3%, which is economically significant (recall that the 10-year cash-out probability for a public firm is zero). For higher risk aversion ($\gamma = 2$), the private leverage ratio is only 49.4%, significantly smaller than 71.3% for the setting with the default option only. Note the significant substitution effect between risky debt and public equity as channels to diversify the entrepreneur's idiosyncratic risk. With public equity, the entrepreneur substitutes away from the risky debt towards public equity to diversify his idiosyncratic risk.

Next, we introduce tax benefits into the model by setting $\tau_e = \tau_m$, but still assume $\tau_g = 0$. For $\gamma = 1$, the option to cash out lowers the initial coupon to $b = 0.54$ from $b = 0.68$ for the firm that has the default option only. The private leverage ratio L_0 at issuance is 55.3%, with a credit spread 138 basis points, compared to $L_0 = 67.9\%$ and a credit spread at 159 basis points when the firm only has the default option. For higher risk aversion ($\gamma = 2$), the private leverage ratio is 67.7%, and initial coupon $b = 0.66$, smaller than $L_0 = 81.4\%$ and $b = 0.85$ for the setting with default option only. While the cash-out option lowers the 10-year default probability from 22.3%

to 9.8%, the significant 10-year cash-out probability (26.8%) and the call-back feature of the debt at the cash-out boundary reduce the differences in credit spreads in the case with and without the cash-out option (186 versus 213 basis points). The table provides compelling results that the option to cash out through external equity is a less effective substitute for the risky debt for diversification purposes when debt has tax benefits.

In the presence of capital gains taxes with $\tau_g = 0.10$, the value of the cash-out option drops. Table 3 shows that the 10-year cash-out probability decreases, and the entrepreneur takes on more debt in order to diversify idiosyncratic risks. However, the quantitative effect is small in our numerical example. We may understand the intuition from the value-matching condition (19). At the cash-out threshold y_u , the entrepreneur obtains less value $(1 - \tau_g)V^*(y_u)$, but enjoys tax rebate $\tau_g(K + I)$. Thus, these two effects partially offset each other, making the effect of capital gains taxes small.

5.2 Idiosyncratic risk, leverage, and risk premia

In this subsection, we study how idiosyncratic volatility affects leverage and risk premia. Figure 3 shows its effect on leverage. As is well known in the complete markets model (e.g., Leland (1994)), an increase in (idiosyncratic) volatility raises default risk, hence the market leverage ratio and the coupon rate for the public firm decrease with idiosyncratic volatility. By contrast, facing incomplete markets, risk averse entrepreneurs will take on more debt to diversify their idiosyncratic risk when idiosyncratic volatility is higher. This result implies that the private leverage ratio for entrepreneurial firms increases with idiosyncratic volatility.

[Insert Figure 3 here.]

We next study the impact of idiosyncratic volatility on the risk premium that the entrepreneur demands. We decompose the entrepreneur's risk premium into two components: the systematic risk premium $\pi^s(y)$ and the idiosyncratic risk premium component $\pi^i(y)$. To motivate our analysis, we rewrite the valuation ODE (16) for the entrepreneur's private value of equity $G(y)$ as follows:

$$\pi^s(y) + \pi^i(y) = \frac{(1 - \tau_e)(y - b - w)}{G(y)} + \frac{1}{G(y)} \left(\mu y G'(y) + \frac{\sigma^2 y^2}{2} G''(y) \right) - r, \quad (26)$$

where the systematic risk premium $\pi^s(y)$ and the idiosyncratic risk premium $\pi^i(y)$ are respectively

given by

$$\pi^s(y) = \eta\omega \frac{G'(y)}{G(y)}y = \eta\omega \frac{d \log G(y)}{d \log y}, \quad (27)$$

$$\pi^i(y) = \frac{\gamma r (\epsilon y G'(y))^2}{2 G(y)}. \quad (28)$$

The first term and the second term on the right side of (26) measure the current yield and the expected percentage change for private value of equity $G(y)$, respectively. The sum of these two terms is the total expected return for private value of equity $G(y)$. Subtracting the risk-free rate from the expected return gives the risk premium for $G(y)$. Intuitively, we may interpret this risk premium as the sum of two components: the systematic risk premium $\pi^s(y)$ and the idiosyncratic risk premium $\pi^i(y)$.

The systematic risk premium $\pi^s(y)$ takes the same form as in standard asset pricing models. It is the product of (market) Sharpe ratio η , systematic volatility ω , and the elasticity of $G(y)$ with respect to y , where the elasticity captures the impact of optionality on risk premium. Despite this standard interpretation for the systematic risk premium, it is worth pointing out that $\pi^s(y)$ also indirectly reflects the non-diversifiable idiosyncratic risks that the entrepreneur bears, and risk aversion γ indirectly affects $\pi^s(y)$ through its impact on $G(y)$.

Unlike $\pi^s(y)$, the idiosyncratic risk premium $\pi^i(y)$ given in (28) *directly* depends on risk aversion γ , and $(\epsilon y G'(y))^2$, the conditional (idiosyncratic) variance of the entrepreneur's equity $G(y)$. The conditional (idiosyncratic) variance term reflects the fact that the idiosyncratic risk premium $\pi^i(y)$ is determined by the entrepreneur's precautionary saving demand, which depends on the conditional variance of idiosyncratic risks (Caballero (1991) and Wang (2006)).

[Insert Figure 4 here.]

We examine the behavior of these risk premia at the default and cash-out thresholds in Figure 4. First consider the default threshold. The entrepreneur's equity is a levered position in the firm. When the firm approaches default, the systematic component of the risk premium $\pi^s(y)$ behaves similarly to the standard valuation model. That is, the significant leverage effect around the default boundary implies that the risk premium diverges to infinity when y approaches y_d .

The idiosyncratic risk premium $\pi^i(y)$ behaves quite differently. We can show that

$$\pi^i(y_d) = \lim_{y \rightarrow y_d} \frac{\gamma r (\epsilon y G'(y))^2}{2 G(y)} = \gamma r (\epsilon y_d)^2 G''(y_d).$$

Using $G''(y_d)$ implied by Theorem 1, we may write $\pi^i(y_d) = -2\gamma r(1 - \tau_e)(y_d - b - w)\epsilon^2/\sigma^2$, which is always positive (due to the option value of default, i.e. $y_d < b + w$). That is, the quadratic dependence of the idiosyncratic risk premium $\pi^i(y)$ on $G'(y)$ makes its value finite at the default boundary y_d .

Using the boundary conditions (19)-(20), we deduce that both idiosyncratic and systematic risk premia are finite at the cash-out threshold. Figure 4 indicates that idiosyncratic risk premium peaks up at the cash-out threshold, while systematic risk premia are much smaller. Both idiosyncratic and systematic risk premium may not be monotonic with respect to the revenue y . Systematic risk premia tend to be small when both default and cash-out options are not deep in the money. Idiosyncratic risk premia tend to be small when the firm is close to default.

5.3 Effects of operating leverage

How does operating leverage affect an entrepreneurial firm's financial leverage? Intuitively, operating leverage increases financial distress risk, and thus should limit debt financing. Panel 3 of Table 4 confirms this intuition for the complete-markets case (the limiting case with $\gamma \rightarrow 0$). As the operating cost w increases from 0.2 to 0.4, the 10-year default probability rises from 2.2% to 6.2%, and the firm issues less debt. On the other hand, equity value also decreases because operating costs lower the operating profits. As a result, the effect on financial leverage ratio is ambiguous. In our numerical examples, this ratio increases with operating costs.

[Insert Table 4 Here.]

Our analysis above shows that risky debt has important diversification benefits for entrepreneurial firms. This effect may dominate the preceding "crowding-out" effect of operating leverage. Table 4 confirms this intuition. As w increases from 0.2 to 0.4, an entrepreneur with $\gamma = 1$ raises debt with increased coupon payments from 0.59 to 0.62. However, the market value of debt decreases because both the 10-year default probability and the cash-out probability increase with w . The private equity value also decreases with w and this effect dominates the decrease in debt. Thus, the private leverage ratio rises with operating costs. This result also holds true for a more risk-averse entrepreneur with $\gamma = 2$. Note that the more risk-averse entrepreneur relies more on risky debt to diversify risk. As a result, the 10-year default probability increases substantially from 26.9% to 50.6% for $\gamma = 2$. But the 10-year cash-out probability decreases from 23.7% to 22.3%.

6 Project choice: Asset substitution versus risk sharing

So far, we have focused on the entrepreneur's financing decisions assuming that the entrepreneur has made the investment in the project. We now turn to the entrepreneur's project choice decision. Jensen and Meckling (1976) point out that there is an important incentive problem associated with debt. They argue that after debt is in place, managers has an incentive to take very risky projects due to the convex feature of equity payoffs. We argue that risk aversion and precautionary saving demand substantially mitigates this asset substitution effect and may overturn the well known Jensen and Meckling risk shifting argument. Empirically, there is little evidence in support of risk shifting incentives.²²

[Insert Figure 5 Here.]

To illustrate the effect of risk aversion/precautionary saving motive on the entrepreneurial risk taking incentive, we consider the following project choice problem. Suppose the risk-averse entrepreneur decides to choose among a continuum of mutually exclusive projects with different idiosyncratic volatilities ϵ in the interval $[\epsilon_{\min}, \epsilon_{\max}]$ after debt is in place. Let F_0 be the market value of existing debt with the coupon payment b . The entrepreneur then chooses idiosyncratic volatility $\epsilon^+ \in [\epsilon_{\min}, \epsilon_{\max}]$ to maximize his own utility. As shown in Section 3, the entrepreneur effectively chooses ϵ^+ to maximize his private value of equity $G(y_0)$, taking the debt contract (b, F_0) as given. Let this maximized value to be $G^+(y_0)$. Naturally, in equilibrium, the lender anticipates the entrepreneur's *ex post* incentive of choosing the level of idiosyncratic volatility ϵ^+ to maximize $G(y_0)$, and prices the initial debt contract accordingly in the competitive capital markets. Therefore, the entrepreneur *ex ante* maximizes the private value of the firm, $S(y_0) = G^+(y_0) + F_0$, taking the (competitive market) debt pricing into account. Note that the solution of this joint investment and financing problem is a fixed point problem.

Figure 5 illustrates the solution of this optimization problem. When $\gamma \rightarrow 0$, the entrepreneur chooses the highest idiosyncratic volatility project with $\epsilon_{\max} = 0.35$. The optimal coupon payment is 0.297. In this case, the entrepreneur effectively faces effectively complete markets. Thus, the Jensen and Meckling (1976) argument applies because equity is a redundant asset under complete markets, the option analogy is appropriate, and the asset substitution problem arises. When the

²²See Graham and Harvey (2001), Rauh (2007), and other papers (Weisbach?).

entrepreneur is risk averse with high γ , his private value of equity $G(y_0)$ becomes decreasing with idiosyncratic volatility ϵ . The intuition is that the entrepreneur discounts his private value of equity because he bears non-diversifiable idiosyncratic risks. As a result, the entrepreneur chooses the project with the lowest idiosyncratic volatility $\epsilon_{\min} = 0.05$, with the corresponding optimal coupon payment being 0.491 (for $\gamma = 1.0$). The increase in debt reflects the fact that the more risk-averse entrepreneur issues more risky debt to diversify risk. From this numerical example, we find that the risk sharing incentive in our model may dominate the asset substitution incentive pointed out by Jensen and Meckling (1976) and thus overturns the Jensen and Meckling result. Even when risk aversion is low (i.e. $\gamma = 0.1$, with an implied idiosyncratic risk premium of 2 bp for $\epsilon = 0.05$, 20 bp for $\epsilon = 0.20$), we still find that the risk aversion effect dominates the risk shifting incentive. This suggests that we do not find much empirical evidence supporting asset substitution may be simply due to entrepreneurial risk aversion/precautionary saving. For publicly traded firms, provided that managerial compensation is tied to firm performance and managers are not fully diversified, the same argument applies because managers are maximizing their own utility.

7 Conclusion

We develop an intertemporal model of entrepreneurial finance and project choice. Unlike publicly held firms, entrepreneurs face significant non-diversifiable idiosyncratic risks. We show that more risk-averse entrepreneurs have lower debt capacity since they default earlier, yet still choose a higher leverage. Cash-out option is an alternative channel for diversification, which reduces the attractiveness of leverage. We also characterize the idiosyncratic risk premium for entrepreneurs, which behaves quite differently compared to the systematic risk premium. Finally, we show that entrepreneur risk aversion may dominate risk-seeking incentives: only those entrepreneurs with very low risk aversion will engage in asset substitution.

Our model can be easily adapted to address the impact of un-diversified executives' decisions on a public firm's capital structure and investment decision. We should point that we do not model the fundamental frictions causing markets to be incomplete. We view endogenous incomplete markets as a complementary perspective, which can have fundamental impacts on questions such as promotion of entrepreneurship and contract design. We leave these important questions for future research.

Appendices

A Market Valuation and Capital Structure of a Public Firm

Well-diversified owners of a public firm face complete markets. Given the Sharp ratio η of the market portfolio and the riskfree rate r , there exists a unique stochastic discount factor (SDF) $(\xi_t : t \geq 0)$ satisfying (see Duffie (2001)):

$$d\xi_t = -r\xi_t dt - \eta\xi_t dB_t, \quad \xi_0 = 1. \quad (\text{A.1})$$

Using this SDF, we can derive the market value of the unlevered firm, $A(y)$, the market value of equity, $E(y)$, and the market value of debt $D(y)$. The market value of the firm is equal to the sum of equity value and debt value:

$$V(y) = E(y) + D(y). \quad (\text{A.2})$$

By the Girsanov theorem, under the risk-neutral probability measure Q , the standard Brownian motion B^Q satisfies $dB_t^Q = dB_t + \eta dt$ (see Duffie (2001)). We then rewrite the dynamics of the cash flows (4) as follows:

$$dy_t = \nu y_t dt + \omega y_t dB_t^Q + \epsilon y_t dZ_t, \quad (\text{A.3})$$

where ν is the risk-adjusted drift defined by $\nu \equiv \mu - \omega\eta$.

Using (A.3), we can derive valuation equations for firm securities by the standard asset pricing theory. We start with the after-tax unlevered firm value $A(y)$. It satisfies the following differential equation:

$$rA(y) = (1 - \tau_m)(y - w) + \nu y A'(y) + \frac{1}{2}\sigma^2 y^2 A''(y). \quad (\text{A.4})$$

This is a second-order ordinary differential equation (ODE). We need two boundary conditions to obtain a solution. One boundary condition describes the behavior of $A(y)$ when $y \rightarrow \infty$. This condition must rule out speculative bubbles. To ensure $A(y)$ is finite, we assume $r > \nu$ throughout the paper. The other boundary condition is related to abandonment. As in the standard option exercise models, the firm is abandoned whenever the cash flow process hits a threshold value y_a for the first time. At the threshold y_a , the following value-matching condition is satisfied

$$A(y_a) = 0, \quad (\text{A.5})$$

because we normalize the outside value to zero. For the abandonment threshold y_a to be optimal, the following smooth-pasting condition must also be satisfied

$$A'(y_a) = 0. \quad (\text{A.6})$$

Solving equation (A.4) and using the no-bubble condition and boundary conditions (A.5)-(A.6), we obtain equation (6) and

$$y_a = \frac{r - \nu}{r} \frac{\theta_1}{\theta_1 - 1} w, \quad (\text{A.7})$$

where

$$\theta_1 = -\sigma^{-2} (\nu - \sigma^2/2) - \sqrt{\sigma^{-4} (\nu - \sigma^2/2)^2 + 2r\sigma^{-2}} < 0. \quad (\text{A.8})$$

We next turn to the valuation of levered firms. First, consider the market value of equity. Let T_d denote the random default time and y_d be the corresponding default threshold. After default, equity is worthless, in that $E(y) = 0$ for $y \leq y_d$. This gives us the value matching condition $E(y_d) = 0$. Before default, equity value $E(y)$ satisfies the following differential equation:

$$rE(y) = (1 - \tau_m)(y - b - w) + \nu y E'(y) + \frac{1}{2} \sigma^2 y^2 E''(y), \quad y \geq y_d. \quad (\text{A.9})$$

When $y \rightarrow \infty$, $E(y)$ also satisfies a no-bubble condition. Solving this ODE and using the boundary conditions, we obtain equation (7). Using the smooth-pasting condition,

$$\left. \frac{\partial E(y)}{\partial y} \right|_{y=y_d} = 0,$$

we obtain the optimal default threshold given in (9).

Similarly, the market value of debt before default satisfies the following differential equation:

$$rD(y) = b + \nu y D'(y) + \frac{1}{2} \sigma^2 y^2 D''(y), \quad y \geq y_d. \quad (\text{A.10})$$

The value-matching condition is given by:

$$D(y_d) = \alpha A(y_d). \quad (\text{A.11})$$

We also imposes a no bubble condition when $y \rightarrow \infty$. Solving yields:

$$D(y) = \frac{b}{r} - \left[\frac{b}{r} - \alpha A(y_d) \right] \left(\frac{y}{y_d} \right)^{\theta_1}, \quad (\text{A.12})$$

Using equation (A.2), we can derive equation (8). Substituting (9) into (8) and using the first-order condition:

$$\frac{\partial V(y_0)}{\partial b} = 0, \quad (\text{A.13})$$

we obtain the optimal coupon rate b^* as a function of y_0 .

Now consider the special case without operating cost (i.e. $w = 0$). We have an explicit expression for debt coupon:

$$b^* = y_0 \frac{r}{r - \nu} \frac{\theta_1 - 1}{\theta_1} \left(1 - \theta_1 - \frac{(1 - \alpha)(1 - \tau_m)\theta_1}{\tau_m} \right)^{1/\theta_1}. \quad (\text{A.14})$$

We also verify that the second order condition is satisfied. Substituting (9) and (A.14) into (8), we obtain the following expression for $V^*(y)$, firm value when debt amount is optimally chosen:

$$V^*(y) = \left[1 - \tau_m + \tau_m \left(1 - \theta_1 - \frac{(1 - \alpha)(1 - \tau_m)\theta_1}{\tau_m} \right)^{1/\theta_1} \right] \frac{y}{r - \nu}. \quad (\text{A.15})$$

Note that this firm value formula only applies at the moment of debt issuance.

B Proof of Theorem 1

First, consider the entrepreneur's optimality after he exits. The entrepreneur solves the standard complete-markets consumption/portfolio choice problem (Merton (1971)). His wealth follows from the following dynamics

$$dx_t = (r(x_t - \phi_t) - c_t) dt + \phi_t (\mu_p dt + \sigma_p dB_t). \quad (\text{B.1})$$

The explicit value function is given by (12). The consumption and portfolio rules are given by

$$\bar{c}(x) = \left(x + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2} \right), \quad (\text{B.2})$$

$$\bar{\phi}(x) = \frac{\eta}{\gamma r \sigma_p}. \quad (\text{B.3})$$

Now to the entrepreneur's decision making while he run the private firm. Before exiting from his business, the entrepreneur's financial wealth evolves as follows:

$$dx_t = (r(x_t - \phi_t) + (1 - \tau_e)(y - b - w) - c_t) dt + \phi_t (\mu_p dt + \sigma_p dB_t), \quad (\text{B.4})$$

for $t < \min(T_d, T_u)$, where T_d is the default time and T_u is the cash-out time. The entrepreneur receives $(1 - \tau_e)(y_t - b - w)$ from his business in flow terms after servicing the debt and paying taxes.

Using the principle of optimality, we claim that the value function $J^s(x, y)$ satisfies the following HJB equation:

$$\begin{aligned} \delta J^s(x, y) = & \max_{c, \phi} u(c) + (rx + \phi(\mu_p - r) - c + (1 - \tau_e)(y - b - w)) J_x^s(x, y) \\ & + \mu y J_y^s(x, y) + \frac{(\sigma_p \phi)^2}{2} J_{xx}^s(x, y) + \frac{\sigma^2 y^2}{2} J_{yy}^s(x, y) + \phi \sigma_p \omega y J_{xy}^s(x, y). \end{aligned} \quad (\text{B.5})$$

The first-order conditions (FOC) for consumption c and portfolio allocation ϕ are as follows:

$$u'(c) = J_x^s(x, y), \quad \phi = \frac{-J_x^s(x, y)}{J_{xx}^s(x, y)} \left(\frac{\mu_p - r}{\sigma_p^2} \right) + \frac{-J_{xy}^s(x, y) \omega y}{J_{xx}^s(x, y) \sigma_p}, \quad (\text{B.6})$$

Conjecture that the value function $J^s(x, y)$ is exponential in wealth and is given by equation (13), where $G(y)$ is a function to be determined. Under the conjectured value function (13), we show that the optimal consumption rule and the portfolio rule are given by (14) and (15), respectively. Substituting these expressions back into the HJB equation (B.5) gives the differential equation (16) for $G(y)$.

We now turn to boundary conditions. First, consider the lower default boundary. At the instant of default, the entrepreneur walks away from his firm's liability and the lender liquidates the firm's asset. The entrepreneur's financial wealth x does not change immediately after default, in that $x_{T_d} = x_{T_d-}$. In addition, the entrepreneur's value function should remain unchanged at the moment of default. That is, the following value-matching condition holds along the default boundary as in standard option exercising problems:

$$J^s(x, y) = J^e(x). \quad (\text{B.7})$$

The above equation implicitly defines the lower default boundary for cash flow y as a function of wealth x : $y = y_d(x)$. Note that in general, the default boundary depends on the entrepreneur's wealth level. Because the default boundary is optimally chosen, the following smooth-pasting conditions at $y = y_d(x)$ must be satisfied:²³

$$\frac{\partial J^s(x, y)}{\partial x} = \frac{\partial J^e(x)}{\partial x}, \quad (\text{B.8})$$

$$\frac{\partial J^s(x, y)}{\partial y} = \frac{\partial J^e(x)}{\partial y}. \quad (\text{B.9})$$

The first smooth-pasting condition (B.8) states that the marginal change in cash flow y has the same marginal effect on the entrepreneur's value functions just before and immediately after defaulting

²³See Krylov (1980), Dumas (1991), and Dixit and Pindyck (1994).

on the firm's liability. Similarly, the second smooth-pasting (B.9) states that the marginal effect of wealth x must be the same on the entrepreneur's value functions just before and immediately after defaulting on the firm's liability. Unlike the complete-markets endogenous default models (Leland (1994)), the entrepreneur's financial wealth x enters as an additional state variable, which gives rise to the second smooth-pasting condition.

We next turn to the upper cash-out boundary. At the instant of cashing out, the entrepreneur sells his firm to well diversified investors and collects firm value $V^*(y)$ given in (8). Since the entrepreneur needs to pay the fixed cost K , retire debt at par F_0 , and pay capital gains taxes, his wealth x_{T_u} immediately after cashing out satisfies

$$x_{T_u} = x_{T_u^-} + V^*(y_{T_u}) - F_0 - K - \tau_g(V^*(y_{T_u}) - K - I).$$

Similar to the lower default boundary, the entrepreneur's value function must satisfy the following value-matching condition:

$$J^s(x, y) = J^e(x + V^*(y) - F_0 - K - \tau_g(V^*(y) - K - I)). \quad (\text{B.10})$$

This equation implicitly determines the upper cash-out boundary $y_u(x)$. Using the same arguments as those for the lower default boundary, the entrepreneur's optimality implies the following smooth-pasting conditions at $y = y_u(x)$:

$$\frac{\partial J^s(x, y)}{\partial x} = \frac{\partial J^e(x + V^*(y) - F_0 - K - \tau_g(V^*(y) - K - I))}{\partial x}, \quad (\text{B.11})$$

$$\frac{\partial J^s(x, y)}{\partial y} = \frac{\partial J^e(x + V^*(y) - F_0 - K - \tau_g(V^*(y) - K - I))}{\partial y}. \quad (\text{B.12})$$

Using the conjectured value function (13), we show that the default and cash-out boundaries $y_d(x)$ and $y_u(x)$ are independent of wealth. We thus simply use y_d and y_u to denote the default and cash-out thresholds, respectively. Using the value matching and smooth pasting conditions (B.7)-(B.9) at y_d , we obtain (17) and (18). Similarly, using the value matching and smooth pasting conditions (B.10)-(B.12) at y_u , we have (19) and (20). Q.E.D.

C Market Value of the Entrepreneurial Firm's Debt

When the entrepreneur neither defaults nor cashes out, the market value of his debt $F(y)$ satisfies the following ODE:

$$rF(y) = b + \nu y F'(y) + \frac{1}{2} \sigma^2 y^2 F''(y), \quad y_d \leq y \leq y_u. \quad (\text{C.1})$$

Note that the lender pays interest income tax at rate τ_i and discounts the after-tax cash flow $(1 - \tau_i)b$ at the after-tax interest rate (after adjusting for systematic risk). At the default trigger y_d , debt recovers the fraction α of after-tax unlevered firm value, in that $F(y_d) = \alpha A(y_d)$. At the cash-out trigger y_u , debt is retired and recovers its face value, in that $F(y_u) = F_0$. Solving (C.1) subject to the boundary conditions gives

$$F(y) = \frac{b}{r} + \left(F_0 - \frac{b}{r}\right) \bar{q}(y) + \left[\alpha A(y_d) - \frac{b}{r}\right] \underline{q}(y), \quad (\text{C.2})$$

where

$$\bar{q}(y) = \frac{y^{\theta_1} y_d^{\theta_2} - y^{\theta_2} y_d^{\theta_1}}{y_u^{\theta_1} y_d^{\theta_2} - y_u^{\theta_2} y_d^{\theta_1}}, \quad (\text{C.3})$$

$$\underline{q}(y) = \frac{y^{\theta_2} y_u^{\theta_1} - y^{\theta_1} y_u^{\theta_2}}{y_u^{\theta_1} y_d^{\theta_2} - y_u^{\theta_2} y_d^{\theta_1}}. \quad (\text{C.4})$$

Here, θ_1 is given by (A.8) and

$$\theta_2 = -\sigma^{-2} (\nu - \sigma^2/2) + \sqrt{\sigma^{-4} (\nu - \sigma^2/2)^2 + 2r\sigma^{-2}} > 1. \quad (\text{C.5})$$

Equation (C.2) admits an intuitive interpretation. It states that debt value is equal to the present value of coupon payment plus the changes in value when default occurs and when cash-out occurs. Note that $\bar{q}(y_0)$ can be interpreted as the present value of a \$1 if cash-out occurs before default, and $\underline{q}(y_0)$ can be interpreted as the present value of a \$1 if the entrepreneur goes bankrupt before cash-out. Q.E.D.

Table 1: CAPITAL STRUCTURE OF ENTREPRENEURIAL FIRMS: THE CASE OF DEBT FINANCING ONLY.

This table reports the results for the setting where the entrepreneur only has default option to exit from his project. The parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, and $\alpha = 0.6$. These parameters are annualized when applicable. The initial revenue is $y_0 = 1$. We report results for two business income tax rates ($\tau_e = 0, \tau_m(11.29\%)$) and three levels of risk aversion. The case “ $\gamma \rightarrow 0$ ” corresponds to the complete-market (Leland) model.

| | coupon b | public debt F_0 | private equity G_0 | private firm S_0 | private leverage (%) L_0 | credit spread (bp) CS | 10-yr default probability (%) $p_d(10)$ |
|------------------------|---------------|-------------------------|----------------------------|--------------------------|----------------------------------|-------------------------------|---|
| $\tau_e = 0$ | | | | | | | |
| $\gamma \rightarrow 0$ | 0.00 | 0.00 | 33.33 | 33.33 | 0.0 | 0 | 0.0 |
| $\gamma = 1$ | 0.31 | 8.28 | 14.39 | 22.68 | 36.5 | 72 | 0.4 |
| $\gamma = 2$ | 0.68 | 14.66 | 5.89 | 20.55 | 71.3 | 166 | 12.1 |
| $\tau_e = \tau_m$ | | | | | | | |
| $\gamma \rightarrow 0$ | 0.35 | 9.29 | 20.83 | 30.12 | 30.9 | 75 | 0.3 |
| $\gamma = 1$ | 0.68 | 14.85 | 7.02 | 21.86 | 67.9 | 159 | 9.5 |
| $\gamma = 2$ | 0.85 | 16.50 | 3.77 | 20.27 | 81.4 | 213 | 22.3 |

Table 2: DECOMPOSITION OF PRIVATE LEVERAGE FOR ENTREPRENEURIAL FIRMS

This table compares a private firm owned by a risk averse entrepreneur with a public firm that has the same amount of debt outstanding (coupon is fixed at $b = 0.85$). There is no option to cash out. The model parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$ and $\tau_e = \tau_m$. These parameters are annualized when applicable. All the results are for initial revenue $y_0 = 1$.

| | 10-yr default probability (%) | public debt | equity value | firm value | financial leverage (%) | credit spread (bp) |
|---|----------------------------------|----------------|-----------------|---------------|---------------------------|-----------------------|
| | $p_d(10)$ | F_0 | G_0 | S_0 | L_0 | CS |
| $\gamma = 2$ ($b = 0.85, y_d = 0.47$) | 22.3 | 16.50 | 3.77 | 20.27 | 81.4 | 213 |
| Public ($b = 0.85, y_d = 0.47$) | 22.3 | 16.50 | 11.10 | 27.60 | 59.8 | 213 |
| Public ($b = 0.85, y_d = 0.35$) | 9.8 | 17.71 | 11.56 | 29.26 | 60.5 | 178 |
| Public ($b = 0.35, y_d = 0.14$) | 0.3 | 9.29 | 20.82 | 30.11 | 30.9 | 75 |

Table 3: CAPITAL STRUCTURE OF ENTREPRENEURIAL FIRMS: THE CASE OF DEBT AND EQUITY FINANCING

This table reports the results for the setting where the entrepreneur has both default and cash-out options to exit from his project. The parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, $I = 10$, and $K = 27$. The initial revenue is $y_0 = 1$. We report results for two business income tax rates ($\tau_e = 0, \tau_m(11.29\%)$), two capital gain tax rates ($\tau_g = 0, 10\%$), and two levels of risk aversion ($\gamma = 1, 2$). The case “ $\gamma \rightarrow 0$ ” corresponds to the complete-market model, where the “cash-out” option effectively allows the firm to adjust leverage once.

| | coupon b | public debt F_0 | private equity G_0 | private firm S_0 | private leverage (%) L_0 | credit spread (bp) CS | 10-yr default probability (%) $p_d(10)$ | 10-yr cash-out probability (%) $p_u(10)$ |
|----------------------------------|---------------|-------------------------|----------------------------|--------------------------|----------------------------------|-------------------------------|---|--|
| $\tau_e = 0, \tau_g = 0$ | | | | | | | | |
| $\gamma = 1$ | 0.11 | 3.20 | 19.95 | 23.14 | 13.8 | 32 | 0.0 | 12.3 |
| $\gamma = 2$ | 0.42 | 10.11 | 10.36 | 20.47 | 49.4 | 115 | 1.9 | 24.1 |
| $\tau_e = 0, \tau_g = 10\%$ | | | | | | | | |
| $\gamma = 1$ | 0.12 | 3.61 | 19.36 | 22.97 | 15.7 | 35 | 0.0 | 9.2 |
| $\gamma = 2$ | 0.43 | 10.36 | 10.01 | 20.36 | 50.9 | 117 | 2.2 | 20.0 |
| $\tau_e = \tau_m, \tau_g = 0$ | | | | | | | | |
| $\gamma = 1$ | 0.54 | 12.29 | 9.92 | 22.22 | 55.3 | 138 | 3.9 | 15.5 |
| $\gamma = 2$ | 0.66 | 13.57 | 6.47 | 20.04 | 67.7 | 186 | 9.8 | 26.8 |
| $\tau_e = \tau_m, \tau_g = 10\%$ | | | | | | | | |
| $\gamma \rightarrow 0$ | 0.35 | 9.29 | 20.83 | 30.12 | 30.9 | 75 | 0.3 | 0.0 |
| $\gamma = 1$ | 0.55 | 12.45 | 9.57 | 22.02 | 56.5 | 138 | 4.2 | 12.3 |
| $\gamma = 2$ | 0.66 | 13.68 | 6.24 | 19.92 | 68.7 | 185 | 10.1 | 23.3 |

Table 4: THE EFFECTS OF OPERATING LEVERAGE: THE CASE OF DEBT AND EQUITY FINANCING

This table reports the results for the setting where the entrepreneur has both default and cash-out options to exit from his project. The parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, $\tau_e = \tau_m$, $\tau_g = 10\%$, $I = 10$, and $K = 27$. The initial revenue is $y_0 = 1$. We report results for two levels of risk aversion ($\gamma = 1, 2$) alongside the complete-market solution ($\gamma \rightarrow 0$).

| | coupon b | public debt F_0 | private equity G_0 | private firm S_0 | private leverage (%) L_0 | credit spread (bp) CS | 10-yr default probability (%) $p_d(10)$ | 10-yr cash-out probability (%) $p_u(10)$ |
|------------------------|---------------|-------------------------|----------------------------|--------------------------|----------------------------------|-------------------------------|---|--|
| $\gamma \rightarrow 0$ | | | | | | | | |
| $w = 0.2$ | 0.35 | 8.03 | 16.73 | 24.76 | 32.4 | 132 | 2.2 | 0.0 |
| $w = 0.4$ | 0.33 | 6.72 | 13.40 | 20.12 | 33.4 | 194 | 6.2 | 0.0 |
| $\gamma = 1$ | | | | | | | | |
| $w = 0.2$ | 0.59 | 10.94 | 6.34 | 17.28 | 63.3 | 237 | 14.1 | 13.1 |
| $w = 0.4$ | 0.62 | 9.41 | 3.98 | 13.39 | 70.3 | 356 | 28.4 | 13.5 |
| $\gamma = 2$ | | | | | | | | |
| $w = 0.2$ | 0.73 | 11.95 | 3.57 | 15.53 | 77.0 | 315 | 26.9 | 23.7 |
| $w = 0.4$ | 0.84 | 10.48 | 1.57 | 12.05 | 86.9 | 503 | 50.6 | 22.3 |

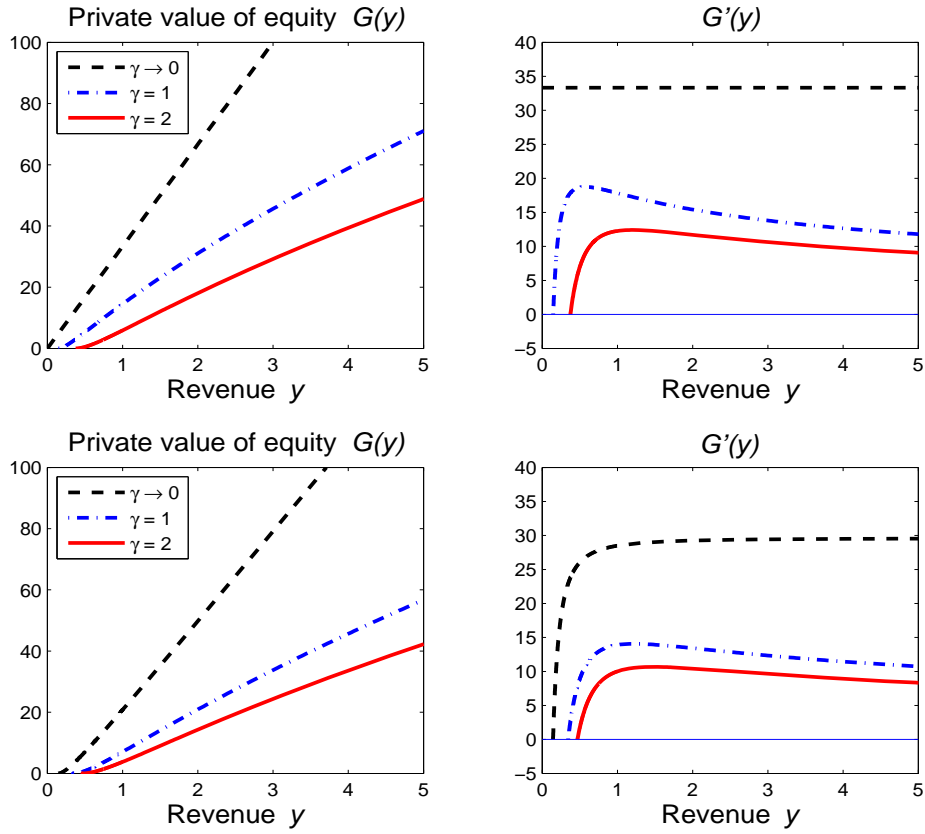


Figure 1: **Private value of equity $G(y)$ as functions of revenue y : the case of debt financing only.** The top two panels plot $G(y)$ and its first derivative $G'(y)$ for $\tau_e = 0$. The bottom two panels plot $G(y)$ and $G'(y)$ for $\tau_e = \tau_m$. In each case, we plot the results for two levels of risk aversion ($\gamma = 1, 2$) alongside the benchmark complete-market solution ($\gamma \rightarrow 0$). The remaining parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, and $\tau_m = 11.29\%$.

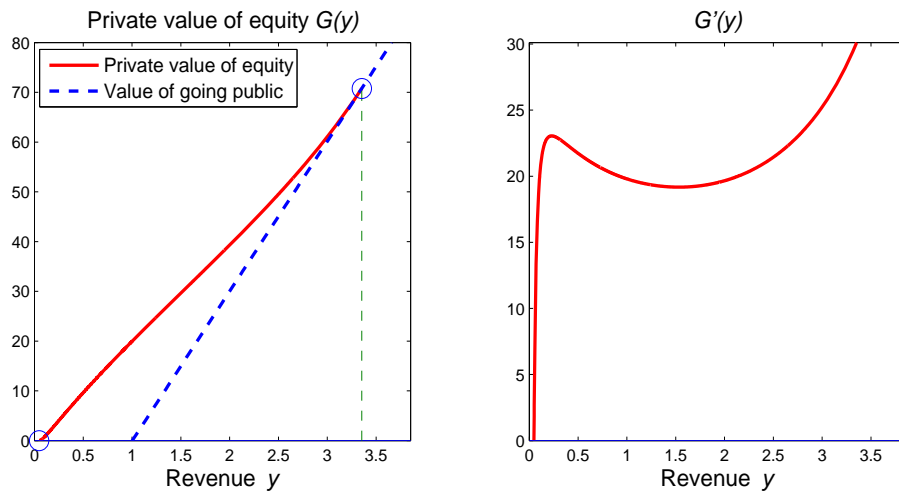


Figure 2: **Private value of equity $G(y)$ as functions of revenue y : the case of debt and equity financing.** We plot the results with the following parameters: $\gamma = 1$, $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\varepsilon = 0.2$, $\alpha = 0.6$, $\tau_e = 0$, $\tau_m = 11.29\%$, $\tau_g = 0$, $I = 10$, and $K = 27$.

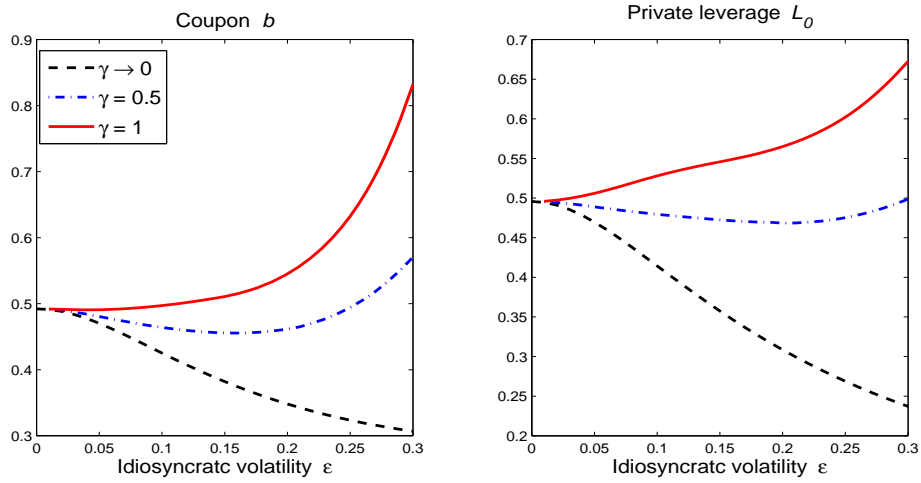


Figure 3: **Comparative statics – optimal coupon and private leverage with respect to idiosyncratic volatilities ϵ : the case of debt and equity financing.** The two panels plot the optimal coupon b and the corresponding optimal private leverage L_0 at $y_0 = 1$. In each case, we plot the results for two levels of risk aversion ($\gamma = 1, 2$) alongside the benchmark complete-market solution ($\gamma \rightarrow 0$). The remaining parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\alpha = 0.6$, $\tau_e = \tau_m = 11.29\%$, $\tau_g = 10\%$, $I = 10$, and $K = 27$.

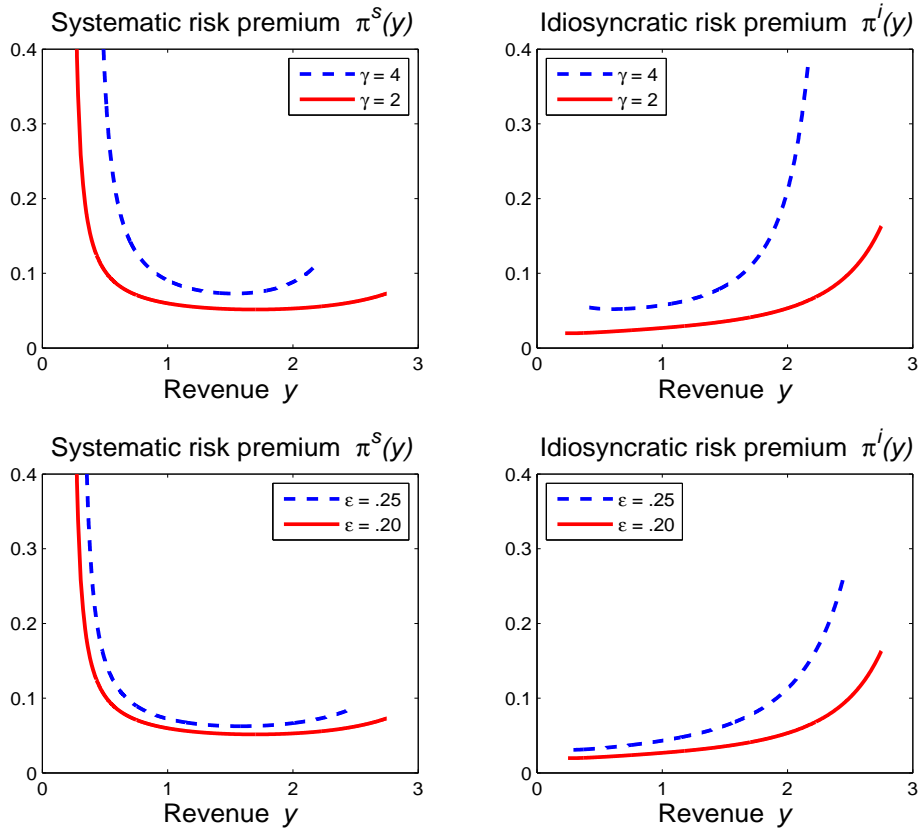


Figure 4: **Systematic and idiosyncratic risk premia as functions of revenue y : the case of debt and equity financing.** This figure plots the systematic and idiosyncratic risk premia that the entrepreneur demands for holding the inside equity of a private firm. The top panels plot the results for two levels of risk aversion ($\gamma = 2, 4$). The bottom panels plot the results for two levels of idiosyncratic volatility ($\epsilon = 0.20, 0.25$). The remaining parameters are: $\gamma = 2$ (when changing ϵ), $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\epsilon = 0.2$ (when changing γ), $\alpha = 0.6$, $\tau_e = 0$, $\tau_m = 11.29\%$, $\tau_g = 10\%$, $I = 10$, and $K = 27$.

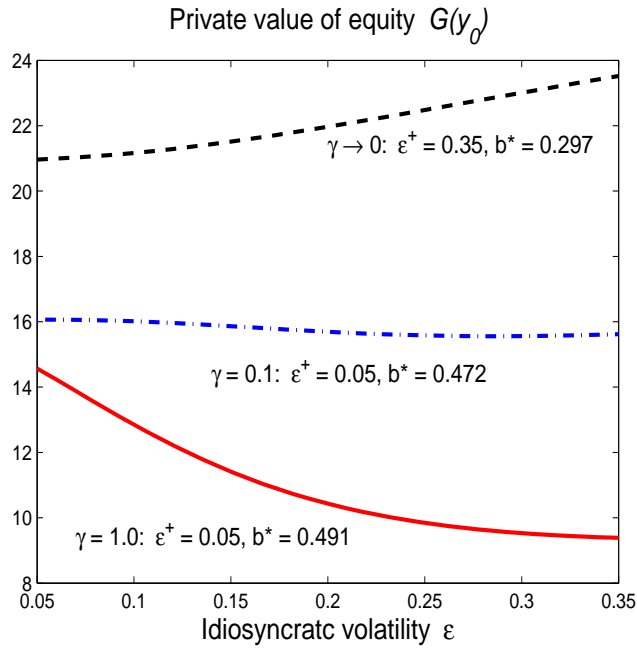


Figure 5: **Private equity value as function of idiosyncratic volatility after optimal debt is in place.** This figure plots the private value of equity for different choices of idiosyncratic volatility ϵ after debt issuance. The coupon is fixed at the optimal value corresponding to given risk aversion. The remaining parameters are: $r = \delta = 0.03$, $\eta = 0.4$, $\mu = 0.04$, $\omega = 0.1$, $\epsilon_L = 0.05$, $\epsilon_L = 0.35$, $\alpha = 0.6$, $\tau_e = \tau_m = 11.29\%$, $\tau_g = 10\%$, $I = 10$, and $K = 27$.

References

- Attanasio, O. P., 1999, “Consumption,” *In: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics, vol. 1B. North-Holland, Amsterdam.*
- Black, F., and J. Cox, 1976, “Valuing Corporate Securities: Some Effects of Bond Indenture Provisions,” *Journal of Finance*, 31, 351–367.
- Cox, J. C., and C. Huang, 1989, “Optimum Consumption and Portfolio Policies When Asset Prices Follow a Diffusion Process,” *Journal of Economic Theory*, 49, 33–83.
- Deaton, A., 1992, *Understanding Consumption*, Oxford University Press, Oxford.
- Dixit, A. K., and R. S. Pindyck, 1994, *Investment Under Uncertainty*, Princeton University Press, Princeton, N.J.
- Duffie, D., 2001, *Dynamic Asset Pricing Theory*, Princeton University Press, Princeton, 3rd edn.
- Duffie, D., and D. Lando, 2001, “Term Structures of Credit Spreads with Incomplete Accounting Information,” *Econometrica*, 69, 633–664.
- Dumas, B., 1991, “Super contact and related optimality conditions,” *Journal of Economic Dynamics and Control*, 4, 675–685.
- Goldstein, R., N. Ju, and H. Leland, 2001, “An EBIT-Based Model of Dynamic Capital Structure,” *Journal of Business*, 74, 483–512.
- Graham, J. R., and C. Harvey, 2001, “The Theory and Practice of Corporate Finance: Evidence from the Field,” *Journal of Financial Economics*, 60, 187–243.
- Hall, R. E., and S. E. Woodward, 2008, “The Burden of the Nondiversifiable Risk of Entrepreneurship,” Working Paper, Stanford University.
- Heaton, J. C., and D. J. Lucas, 2004, “Capital Structure, Hurdle Rates, and Portfolio Choice Interactions in an Entrepreneurial Firm,” Working Paper, Northwestern University.
- Jensen, M. C., and W. H. Meckling, 1976, “Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure,” *Journal of Financial Economics*, 3, 305–360.

- Kimball, M., 1990, "Precautionary saving in the small and in the large," *Econometrica*, 58, 53–73.
- Kimball, M., and N. Mankiw, 1989, "Precautionary saving and the timing of taxes," *Journal of Political Economy*, 97, 863–879.
- Krylov, N., 1980, *Controlled Diffusion Processes*, Springer, Berlin.
- Leland, H., 1994, "Corporate Debt Value, bond covenants, and Optimal Capital Structure," *Journal of Finance*, 49, 1213–1252.
- Leland, H., 1998, "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance*, 53, 1213–1243.
- Leland, H. E., and D. H. Pyle, 1977, "Informational Asymmetries, Financial Structure, and Financial Intermediation," *Journal of Finance*, 32, 371–387.
- Mello, A. S., and J. E. Parsons, 1992, "Measuring the Agency Cost of Debt," *Journal of Finance*, 47, 1887–1904.
- Merton, R. C., 1971, "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*, 3, 373–413.
- Merton, R. C., 1973, "An Intertemporal Capital Asset Pricing Model," *Econometrica*, 41, 867–887.
- Miao, J., and N. Wang, 2007, "Investment, Consumption and Hedging under Incomplete Markets," *Journal of Financial Economics*, 86, 608–642.
- Moskowitz, T. J., and A. Vissing-Jorgensen, 2002, "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?," *American Economic Review*, 92, 745–778.
- Myers, S. C., and N. S. Majluf, 1984, "Corporate Financing and Investment Decisions when Firms Have Information That Investors do not Have," *Journal of Financial Economics*, 13, 187–221.
- Rampini, A. A., 1990, "Entrepreneurial activity, risk, and the business cycle," *Journal of Monetary Economics*, 51, 555–573.
- Schumpeter, J., 1934, *The Theory of Economic Development: An Inquiry into Profits Capital Credit Interest and the Business Cycle*, Cambridge, MA.

Svensson, L., and I. Werner, 1993, “Nontradeable assets in incomplete markets: pricing and portfolio choice,” *European Economic Review*, 37, 1149–1168.

Wang, N., 2003, “Caballero meets Bewley: The permanent-income hypothesis in general equilibrium,” *American Economic Review*, 93, 927–936.

Wang, N., 2006, “Generalizing the Permanent-Income Hypothesis: Revisiting Friedman’s Conjecture on Consumption,” *Journal of Monetary Economics*, 53, 737–752.